The {2,3}-hyperidentities in invertible {2,3}-algebras

Yuri, Movsisyan
Yerevan State University
Yerevan, Armenia
e-mail: yurimovsisyan@yahoo.com

Hegine, Ghumashyan
Yerevan State University
Yerevan, Armenia
e-mail: hgumashyan@yahoo.com

ABSTRACT
In this paper the invertible $\{2,3\}$-algebras with balanced $\{2,3\}$-hyperidentities of the first sort, and the length 4 are characterized.

Keywords
Quasigroup, invertible algebra, hyperidentity, balanced hyperidentity.

1. INTRODUCTION
The following second order formula is called a hyperidentity:

$$\forall X_1, \ldots, X_m \, \forall x_1, \ldots, x_k \; (W_1 = W_2),$$

where $X_1, \ldots, X_m$ are functional and $x_1, \ldots, x_k$ are subject variables in words (terms) W_1, W_2. Usually a hyperidentity is specified without universal quantifiers of the prefix: $W_1 = W_2$. If arities $|X_1| = n_1$, ..., $|X_m| = n_m$, the hyperidentity $W_1 = W_2$ is called an $\{n_1, \ldots, n_m\}$-hyperidentity. The hyperidentity $W_1 = W_2$ is said to be satisfied in the algebra $(Q; \Sigma)$ if this equality holds whenever every functional variable X_i is replaced by an arbitrary operation of the corresponding arity from Σ and every objective variable x_j is replaced by an arbitrary element from Q.

For example, if $Q(\cdot)$ is a distributive quasigroup, then the algebra $Q(\cdot, \backslash, /)$ satisfies the hyperidentities of distributivity:

$$X(x, Y(y, z)) = Y(X(x, y), X(x, z)),$$

$$X(Y(x, y), z) = Y(X(x, z), X(y, z)).$$

Moreover, if $Q(A)$ is a distributive quasigroup, then the algebra $Q\left(A, A^{-1}, {}^{-1}A, {}^{-1}(A^{-1}), ({}^{-1}A)^{-1}, A^*\right)$ satisfies these hyperidentities, where $A^*(x, y) = A(y, x)$,

$$A^{-1}(x, z) = y \longleftrightarrow A(x, y) = z \longleftrightarrow {}^{-1}A(z, y) = x.$$

If $Q(A)$ is a medial quasigroup, then the algebra $Q\left(A, A^{-1}, {}^{-1}A, {}^{-1}(A^{-1}), ({}^{-1}A)^{-1}, A^*\right)$ satisfies the hyperidentity of mediality:

$$X(Y(x, y), Y(u, v)) = Y(X(x, u), X(y, v)).$$

A hyperidentity is called balanced if each subject variable of the hyperidentity occurs in both part only once. A balanced hyperidentity is called of the first sort, if the subject variables in the left and right parts of the equality are ordered identically. The number of subject variables in a balanced hyperidentity is called length of this hyperidentity.

We call the algebra $Q(\Sigma)$ with binary and ternary operations a $\{2,3\}$-algebra. The algebra $Q(\Sigma)$ is called invertible, if $Q(A)$ is a quasigroup (for some arity) [1] for any operation $A \in \Sigma$.

The classification of associative hyperidentities and the criterions of their satisfiability in q-algebras and in e-algebras are brought in ([2],[3],[4]). On the solution of similar problems for hyperidentities of ternary associativity in invertible algebras see ([2]) (also see ([5])).

2. RESULTS AND DISCUSSION
In the present paper the criterions of satisfiability for following balanced $\{2,3\}$-hyperidentities of length 4 and of the first sort in invertible $\{2,3\}$-algebras are investigated:

$$X(Y(x, y, z), u) = X(x, Y(y, z, u)), \tag{1}$$

$$Y(X(x, y), u, v) = Y(x, X(y, u), v), \tag{2}$$

$$Y(X(x, y), u, v) = Y(x, y, X(u, v)), \tag{3}$$

$$X(Y(x, y, z), u) = Y(X(x, y), z, u), \tag{4}$$

$$X(Y(x, y, z)r, u) = Y(x, X(y, z), u), \tag{5}$$

$$X(Y(x, y, z), u) = Y(x, y, X(z, u)). \tag{6}$$

REFERENCES
[1] V.D. Belousov, *n-ary quasigroups*, "Shtiintsa", 1972.

[2] Yu.M. Movsisyan, *Introduction to the theory of algebras with hyperidentities*, Yerevan State University Press, 1986.

[3] Yu.M. Movsisyan, *Hyperidentities and hypervarieties in algebras*, Yerevan State University Press, 1990.

[4] Yu.M.Movsisyan, *Hyperidentities in algebras and varieties. Uspekhi Mat.Nauk 53(1),1998,61-114* (English transl. in Russian Math. Surv. 53(1) (1998),57-108).

[5] J. Usan, *n-groups in the light of the neutral operations*, Electronic version, 2006.

[6] A.I. Maltsev, *Algebraic systems*, Nauka, 1970.