Laplacian Based LF Quality Map for Phase Reconstruction

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ABSTRACT

In this paper a novel method of quality map construction based on Laplacian filtering (LF) of the given wrapped phase data is presented. Experimental results show that the proposed quality map can have higher reliability than conventional quality maps when unwrapping noisy, low-modulation, and/or discontinuous phase maps.

Keywords

Quality map, phase reconstruction, SAR interferometry, image processing.

1. INTRODUCTION

Phase unwrapping is one of the most important processing steps in many advanced imaging applications such as magnetic resonance imaging (MRI) [1, 2], satellite radar interferometry (SAR) [3, 4, 5], and optical interferometry [6], where the required data is encoded in the form of a phase distribution. The (absolute) phase extracted from the actual signal is wrapped into the interval $(-\pi, \pi]$ and called principal or wrapped phase. The relationship between the wrapped phase ψ and the absolute (unwrapped) phase ϕ is stated as

$$\psi = \phi + 2\pi k, \qquad \psi \in (-\pi, \pi]. \tag{1}$$

In the applications mentioned above the wrapped phase is useless until 2π phase discontinuities are removed. This is realized by *phase unwrapping* algorithms. Simply stated, the phase unwrapping problem is to obtain an estimate for the absolute phase from the given wrapped phase values.

Many various approaches to 2-D phase unwrapping have been proposed over the past several decades, but only a limited number are currently in common use. The methods developed for phase unwrapping problem can be roughly separated in two large families: Path-following (local) methods and Minimum-norm (global) methods. The first family of algorithms relies on performing an integration of discrete gradients of wrapped phase along a selected path. The algorithms of the second family rely on a global approximation of the absolute phase. A comprehensive review of this two families of algorithms is given in [7].

Many path-following algorithms rely completely on *quality maps*. Comparison of quality maps with the phase and residue (phase inconsistency) data shows that the corrupted phase (and residue) tends to have low-quality values. This suggests an approach to phase unwrapping known as *quality-guided path-following* in which the integration path follows the high-quality pixels and avoids the low-quality pixels.

Quality maps also play an important role in assigning appropriate weights in some of the Minimum-norm algorithms. Here small weights are assigned to noisy regions to reduce the effect of noise.

To summarize, quality maps play a key role in the phase unwrapping process. Thus, it is an important task to develop a reliable quality map.

2. CONVENTIONAL QUALITY MAPS

Quality maps essentially are arrays of values. They define the quality (goodness) of each pixel of the given phase data. There are many quality maps for guiding path-following phase unwrapping algorithms. The most reliable quality maps are:

- Correlation,
- Pseudo-Correlation,
- Phase Derivative Variance,
- Maximum Phase Gradient,
- Second Difference.

2.1 Correlation

Correlation quality map is specified only for SAR interferometry (InSAR) by the correlation coefficients of the InSAR data. These coefficients are defined to be the magnitudes of the complex-valued InSAR data. The correlation map is important because it is the best estimator of the quality of the phase data extracted from the InSAR data.

For two SAR images $u_{m,n}$ and $v_{m,n}$, the complex-valued InSAR image $z_{m,n}$ is defined by the following normalized sum [7]:

$$z_{m,n} = \frac{\sum u_{i,j} v_{i,j}^*}{\sqrt{\sum |u_{i,j}|^2 \sum |v_{i,j}|^2}}, \tag{2}$$

where $v_{i,j}^*$ is the complex conjugate of $v_{i,j}$. The formation of this sum, called "multilook averaging", is performed in the $k \times k$ neighborhood centered at each pixel (m, n). Correlation quality map is defined by the correlation coefficients, which are the magnitudes of the complex values $z_{m,n}$ in Equation 2:

$$Q_{m,n} = |z_{m,n}|. \tag{3}$$

2.2 Pseudo-Correlation (PSD)

Pseudo-correlation map is identical to the correlation map, except the magnitude of the complex-valued data is assumed to be unity. A pseudo-correlation map can be derived from any phase data. It defines the goodness of each pixel using the following equation:

$$Q_{m,n} = \frac{\sqrt{\left(\sum \cos\psi_{i,j}\right)^2 + \left(\sum \sin\psi_{i,j}\right)^2}}{k^2}, \tag{4}$$

where the sums are evaluated in $k \times k$ neighborhood of each pixel (m,n), and $\psi_{i,j}$ is the wrapped phase value of the pixel (m,n).

In practice it is not such a good estimator of the phase quality, which will be seen in Section 4.

2.3 Phase Derivative Variance (PDV)

The third quality map is the phase derivative variance quality map which shows the statistical variance of the phase derivative. Each value of the PDV quality map indicates the badness, rather than goodness, of the phase data.

The PDV quality map defines the goodness of each pixel by the following equation:

$$Q_{m,n} = \frac{1}{B_{m,n}}, \tag{5}$$

where $Q_{m,n}$ is the quality of the pixel (m,n). $B_{m,n}$ represents the badness of the pixel (m,n), which is given by the following equation:

$$B_{m,n} = \frac{\sqrt{\sum \left(\Delta^x_{i,j} - \overline{\Delta^x}_{m,n}\right)^2} + \sqrt{\sum \left(\Delta^y_{i,j} - \overline{\Delta^y}_{m,n}\right)^2}}{k^2}, \tag{6}$$

where the sums are evaluated in $k \times k$ neighborhood of each pixel (m,n) centered in that pixel. The terms $\Delta^x_{i,j}$ and $\Delta^y_{i,j}$ are wrapped phase derivatives in x and y directions:

$$\Delta^x_{i,j} = W(\psi_{i+1,j} - \psi_{i,j}), \tag{7}$$

$$\Delta^y_{i,j} = W(\psi_{i,j+1} - \psi_{i,j}), \tag{8}$$

where W defines a wrapping operator that wraps values of its argument into the range $(-\pi, \pi]$. The terms $\overline{\Delta^x}_{m,n}$ and $\overline{\Delta^y}_{m,n}$ are the mean values of the derivatives $\Delta^x_{i,j}$ and $\Delta^y_{i,j}$ in a corresponding $k \times k$ window.

2.4 Maximum Phase Gradient (MPG)

The maximum phase gradient quality map measures the magnitude of the largest phase gradient in a $k \times k$ neighborhood of each pixel. The motivation for the definition of MPG quality map is the observation that in noisy phase regions, gradients tend to be large. Like PDV quality map, MPG indicates the badness of the phase data which is given by the following equation:

$$B_{m,n} = \max \left\{ \begin{array}{c} \max_{i,j}\{|\Delta^x_{i,j}\}| \\ \max_{i,j}\{|\Delta^y_{i,j}\}| \end{array} \right\}. \tag{9}$$

The quality of each pixel $Q_{m,n}$ is then calculated using Equation 5.

2.5 Second Difference (SD)

Second Difference quality map also indicates the badness of each pixel, which is defined as follows:

$$B_{m,n} = \sqrt{H^2(i,j) + V^2(i,j)}, \tag{10}$$

where H and V are the horizontal and vertical second differences:

$$H(i,j) = W(\psi_{i-1,j} - \psi_{i,j}) - W(\psi_{i,j} - \psi_{i+1,j}), \tag{11}$$

$$V(i,j) = W(\psi_{i,j-1} - \psi_{i,j}) - W(\psi_{i,j} - \psi_{i,j+1}). \tag{12}$$

The quality $Q_{m,n}$ of each pixel is again calculated using Equation 5.

As it will be shown in Section 4, described quality maps lead to completely different results when used with phase unwrapping algorithms. That is why choosing an appropriate quality map is a very important issue. In many cases selection of the appropriate quality map for the given data depends on the data itself. Experiments show that MPG and PDV quality maps guide the phase unwrapping through better unwrapping paths.

3. PROPOSED LAPLACIAN BASED QUALITY MAP

Let us consider a complex-valued *exp* of Equation 1:

$$e^{i\psi} = e^{i(\phi + 2\pi k)} = e^{i\phi}. \tag{13}$$

As it can be seen from Equation 13, the difference between wrapped and unwrapped phases disappears. However the transformed well-defined complex function called *phasor* $F = e^{i\psi}$ does not contain 2π jumps. Thus, it is convenient to use F to derive a new quality map.

The proposed LF (Laplacian Filtering) quality map is defined by the following equation:

$$B = |\nabla^2 F|, \tag{14}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is a two dimensional Laplacian operator.

Since the input wrapped data represents a set of discrete pixels, the discrete implementation of the proposed quality map should be discussed. It is known from image processing [8], that linear filtering of an image F of size $M \times N$ with a filter mask w of size $m \times n$ is given by the expression:

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) F(x+s, y+t), \tag{15}$$

where $a = (m-1)/2$ and $b = (n-1)/2$, and equation is applied for $x = 1, 2, \ldots, M$ and $y = 1, 2, \ldots, N$. Let us notice that the coefficient $w(0,0)$ coincides with image value F(x,y), indicating that the filter is centered at (x,y).

Laplacian filtering of an image highlights regions of rapid intensity change and is therefore often used for edge detection. Digital implementation of the two-dimensional Laplacian operator $\nabla^2 F$ is obtained by the following equation:

$$\begin{aligned} \nabla^2 F &= [F(x+1,y) + F(x-1,y) \\ &\quad + F(x,y+1) + F(x,y-1)] \\ &\quad - 4F(x,y). \end{aligned} \tag{16}$$

This equation can be implemented using the mask w shown in Figure 1.

w(-1,-1) = 0.1667	w(-1,0) = 0.6667	w(-1,1) = 0.1667
w(0,-1) = 0.6667	w(0,0) = -3.3333	w(0,1) = 0.6667
w(1,-1) = 0.1667	w(1,0) = 0.6667	w(1,1) = 0,1667

Figure 1. Discrete Laplacian Filter.

Like PDV, MPG and SD quality maps, proposed map indicates the badness of the phase data which is given as the absolute value of the Laplacian filter response:

$$B_{m,n} = \sqrt{Re^2[g(x,y)] + Im^2[g(x,y)]}. \qquad (17)$$

The quality $Q_{m,n}$ of each pixel is calculated using Equation 5.

4. EXPERIMENTAL RESULTS

Let us focus on simulated data in order to evaluate the advantages of the proposed quality map. As an accuracy measure we use the root-mean-squared-error: $RMSE = \sqrt{\frac{1}{N_x N_y}\sum\left(\phi(x_s,y_s) - \hat{\phi}(x_s,y_s)\right)^2}$. As a discrete mask for Laplacian filtering we use one presented in Figure 1.

To evaluate the proposed quality map we use the path-following method presented in [9] to unwrap the simulated phase data. Method was extended to use a quality map for guiding the path. As a benchmark for comparison we use the results of phase unwrapping obtained using four conventional quality maps presented in Section 2: pseudo-correlation (PC), maximum phase gradient (MPG), phase derivative variance (PDV), and second difference (SD) quality map.

The Gaussian absolute phase test function, presented in Figure 2 (a), is defined by the following formula considered on the integer grid $x = (-64 : 63), y = (-64 : 63)$:

$$\phi(x,y) = A_\phi e^{-\frac{x^2+y^2}{2\sigma^2}}, \qquad (18)$$

where $A_\phi = 14\pi$ and $\sigma = 17.5$.

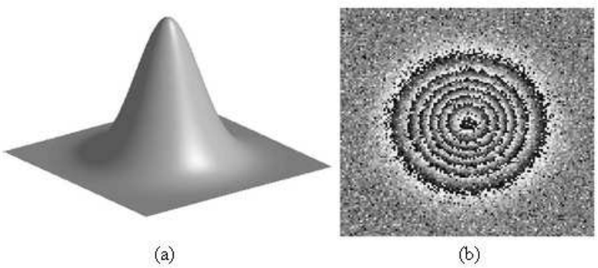

Figure 2: Gaussian phase test function. (a) Original absolute phase, (b) observed wrapped phase with additive white Gaussian noise.

Figure 2 (b) illustrates the noisy wrapped phase data obtained by wrapping the absolute phase and adding white Gaussian noise. Figure 3 presents five quality maps used in the experiment.

Table 1 contains numerical results of the used quality-guided algorithm for different quality maps.

Table 1. Experimental Results

Quality Map	RMSE
PSD	0.828
PDV	0.315
MPG	0.208
SD	5.4825
LF	0.182

Figure 4 illustrates the absolute phase reconstructed by the unwrapping algorithm using four conventional quality maps and the proposed Laplacian based map.

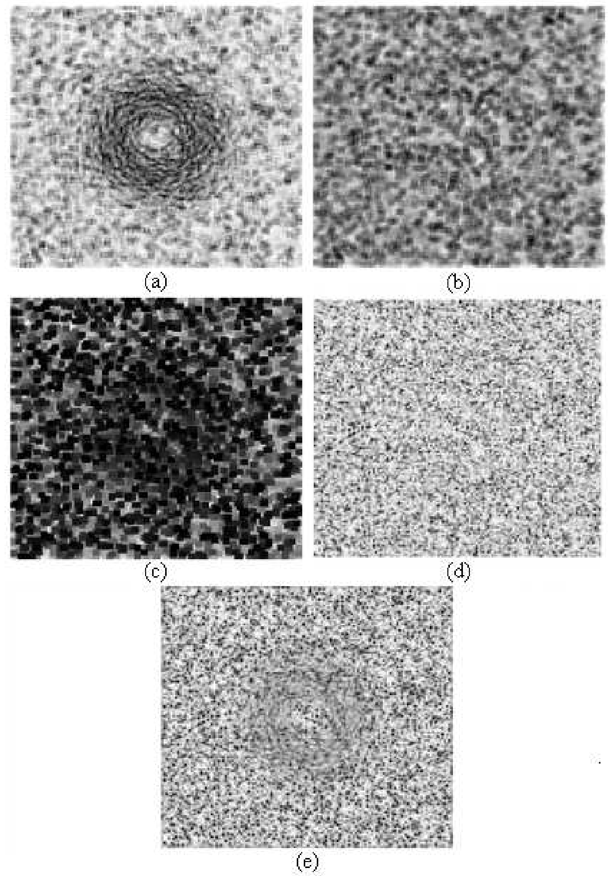

Figure 3: Five quality maps obtained from noisy wrapped phase data. (a) PSD quality map, (b) PDV quality map, (c) MPG quality map, (d) SD quality map, (e) LF quality map.

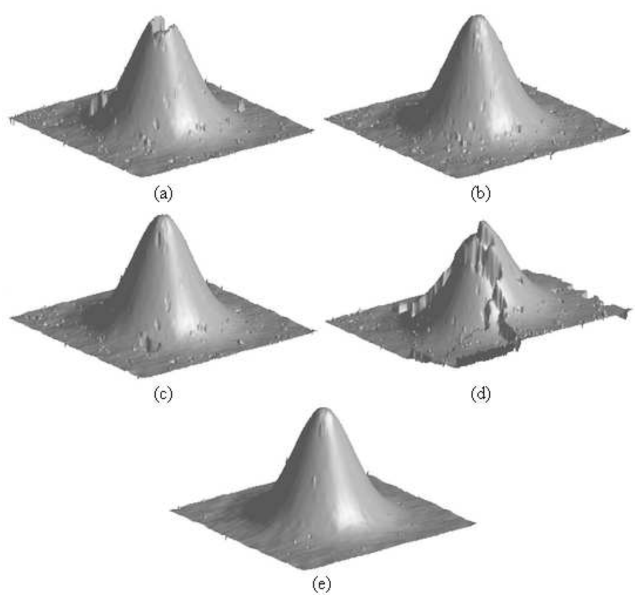

Figure 4: Reconstructed phase images obtained using (a) PSD quality map, (b) PDV quality map, (c) MPG quality map, (d) SD quality map, (e) LF quality map.

The performed experiments show that the used unwrapping algorithm demonstrates the best performance when using the proposed LF quality map. It also can be seen, that among the presented four conventional quality maps, MPG and PDV are the most reliable ones.

5. CONCLUSIONS

In this paper five conventional quality maps were described and an alternative quality map was proposed. The new quality map is based on Laplacian filtering of the wrapped phase data. We have demonstrated that a new quality map for quality-guided path-following phase unwrapping works effectively in comparison with the conventional quality maps. It is shown that the proposed quality map is more reliable than the PDV quality map, which is generally considered as the most reliable measure of phase quality. Hence the proposed LF quality map can be considered as a good phase quality indicator.

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