Numerical calculation of Charm quark mass dependence of the chromomagnetic dipole operator contribution to $\overline{B} \to X_s\gamma$ at $O\left(\alpha_s^2\right)$.

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ABSTRACT

We extend the existing methods of calculation of multiloop Feynman diagram integrals by using the parallel computing methods, which are included in new mathematical packages and are allowed by multi core processors, cluster systems. This letter is explaining the numerical calculation methods which will be programmed for parallel multi core computing. The main methods are the Laporta's algorithm for automatic integral reduction for higher order perturbative calculations, the sector decomposition method for reduction of overlapping infrared divergences, and the Mellin–Barnes representation for the integration of integrals containing $1/(x+y)^{\alpha}$ integrands.

All the descriptions of these methods will be shown on the example of $\overline{B} \to X_s\gamma$ decay at $O\left(\alpha_s^2\right)$. More precisely, on the example of charm quark mass dependence of the interference of electromagnetic and chromomagnetic dipole operators contribution to $\overline{B} \to X_s\gamma$ at $O\left(\alpha_s^2\right)$.

Keywords

Parallel and distributed computing.

1. INTRODUCTION

As a flavour changing neutral current process the inclusive decay $\overline{B} \to X_s\gamma$ is loop-induced and therefore highly sensitive to new degrees of freedom beyond the Standard Model. To tap the full potential of this decay channel in deriving constraints on the parameter space of new physics models both the experiments and the Standard Model calculations should be known as accurately as possible.

On the experimental side, the latest measurements are reported in [1,2] by Belle and BABAR, and the world average performed by the Heavy Flavor Averaging Group [3] for $E_\gamma > 1.6\,\text{GeV}$ reads

$$Br\left(\overline{B} \to X_s\gamma\right) = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4} \qquad (1.1)$$

where the errors are statistical and systematical due to the extrapolation to the common lower-cut in the photon energy, and due to the $\overline{B} \to X_d\gamma$ contamination, respectively.

In order to compete with the given experimental accuracy the theoretical prediction of the $\overline{B} \to X_s\gamma$ branching ratio has to be known at the next-to-next-to-leading order (NNLO) level. There have been great efforts of several groups within the last few years to achieve this goal. The three-loop dipole operator matching was found in [4], the three-loop mixing of the four-quark operators in [5], and the three-loop mixing of the dipole operators was calculated in [6]. Furthermore, the four-loop mixing of the four-quark operators into the dipole operators was calculated in [7]. The two-loop matrix elements of the electromagnetic dipole operator together with the corresponding bremsstrahlung terms can be found in [8-11]. The three-loop matrix elements of the four-quark operators are given in [12] within the so-called large-β_0 approximation. The calculation that goes beyond this approximation by employing an interpolation in the charm quark mass m_c is presented in [13]. The combination of all these individual contributions culminated in a first estimate of the $\overline{B} \to X_s\gamma$ branching ratio at $O\left(\alpha_s^2\right)$ [14]. For $E_\gamma > 1.6\,\text{GeV}$ it reads

$$Br\left(\overline{B} \to X_s\gamma\right) = (3.15 \pm 0.23) \times 10^{-4} \qquad (1.2)$$

Herein, we should mention that there are several perturbative and non-perturbative effects that have not been considered when deriving this estimation.

In the present Letter we calculate the charm quark mass dependence of the (O_7, O_8)-interference contribution to the photon energy spectrum $d\Gamma\left(b \to X_s^{partonic}\gamma\right)/dE_\gamma$ and the total decay width $\Gamma\left(b \to X_s^{partonic}\gamma\right)$, excluding charm quarks in the final state. The impact of the m_c-dependence on the branching ratio will be taken into account together with other new contributions in the forthcoming analysis.

The organization of this Letter is as follows. In the Section 2 the calculation methods on the example of $\overline{B} \to X_s\gamma$ decay at $O\left(\alpha_s^2\right)$ are fully and precisely described step by step. The second section is fragmented into the calculation procedures and methods used.

2. CALCULATION PROCEDURE AND METHODS

2.1. Feynman diagrams

For the (O_7, O_8) interference of the partonic $b \to s\gamma$ decay we need six Feynman diagrams which are shown in Fig 1.

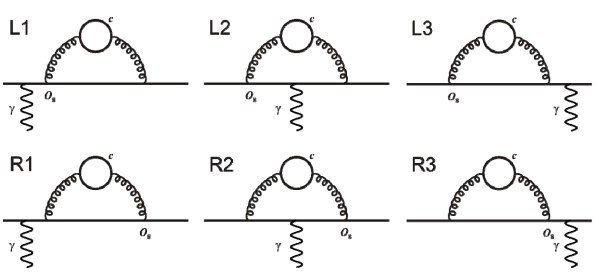

Fig. 1. Diagrams of O_8 chromomagnetic dipole operator contributing to $b \to s\gamma$ decay.

Consider diagram R2 from Fig 1 for a detailed view. All the notations of R2 are shown in Fig 2.

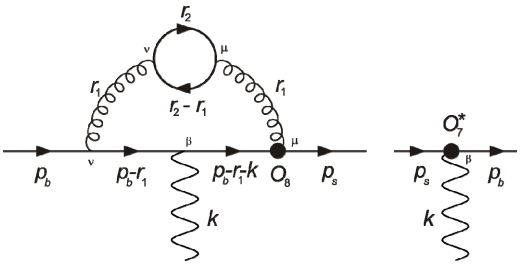

Fig. 2. Diagram R2 from Fig 1. Notations: p_b, p_s, r_1, r_2, k – four momentum of b-quark, s-quark, gluon, c-quark, photon correspondingly. O_8 – chromomagnetic operator, O_7^* – complex conjugate electromagnetic operator.

Using the diagram, we can write the Dirac expression for it. The rest of this section will be devoted to the description of the methods for calculating the Dirac expression.

2.2. Manipulating Dirac expression

The Dirac expression needs to be calculated. To do so we need to bring the expression to the form suitable for integration by the loop integrals.

At first, the numerator of integrand should be simplified. For that we use MatrixExp or Tracer packages.

As a second step we denote the propagators of denominator as follows:

$$\begin{aligned} P1 &= -2p_b r_1 + r_1^2, & P6 &= kr_2, \\ P2 &= -2p_b r_1 + r_1^2 - 2p_b k + 2kr_1, & P7 &= p_b r_2, \\ P3 &= r_1^2, & P8 &= m_b^2 + k^2 - 2p_b k, \\ P4 &= r_2^2 - m_c^2, & P9 &= k^2. \\ P5 &= r_2^2 - 2r_1 r_2 + r_1^2 - mc^2, \end{aligned} \qquad (2.1)$$

Here P1-P5 are the real propagators from the fig. 2 and the P5-P9 are denoted as fictive propagators of some scalar products from the numerator of integrand.

By the use of programs (programmed in Mathematica) we represent the numerator and denominator via (2.1) propagators P1-P9. Part of the result is shown in (2.2).

$$\frac{128P7^2}{P3^2P4P5P8P9} - \frac{64(-3+d)mb^2P7^2}{P1P3^2P4P5P8P9} + etc \qquad (2.2)$$

The loop integrations are still present in (2.2).

2.3. Automatic Integral Reduction (AIR)

Two types of computations are generally required for the evaluation of cross-sections and decay rates: loop integrations over the momenta of virtual particles, and phase-space integrations over the momenta of particles in the final state. At higher orders, in perturbation theory both are hard tasks; this is primarily due to a large number of integrals. Unfortunately, methods for the analytic computation of loop and phase-space integrals are complicated. A solution to this problem is to construct algorithms which reduce the number of integrals to a few master integrals, and directly calculate the master integrals only.

The method of integration by parts (IBP) for the reduction of loop integrals was introduced in [15]. Integrals which have common propagators (or, equivalently, belong to the same topology) satisfy linear algebraic identities. These identities can be derived with the IBP method and can be cleverly combined to produce reduction identities to master integrals. Laporta has improved this method and proposed a fully automated method for the reduction of generic loop amplitudes [16]. The algorithm proposed by Laporta has already been used in a variety of calculations. However its efficient implementation in the computer program is not trivial. The main difficulties arise from the fact that typical multiloop calculations require an enormous number of IBP/LI equations (10^5−10^6). In the process of Gauss elimination the algorithm can produce very large expressions; one must optimize for their efficient manipulation.

Now there is a program written in MAPLE (by Anastasiou, Lazopolous) based on the method of [16], for the Automatic Integral Reduction at higher orders in perturbation theory. The user should supply template IBP/LI equations for the integrals of a topology, optional information on the vanishing integrals of the topology and the master integrals (if known), and a small number of parameters controlling the treatment of large expressions. There is no need for advanced knowledge of the MAPLE platform. The input can be supplied with easy to modify text files, and AIR can be controlled with very simple scripts. But for more efficient use of this algorithm, we suppose to write a new one to parallelise all the processes described above for faster and more efficient use of multi core architectures.

The details of the calculation of the abovementioned example will be shown beneath. The terms of P1-P9 propagators in (2.2) fractions are raised to positive or negative powers, they will be denoted as ν_i. It will be useful to know the values of the parameters ν_i for which the corresponding integrals vanish (tadpoles, scale-less bubbles). This information is not formally required; by solving the IBP equations one will eventually find that tadpoles, etc, are indeed vanishing. However, it is more efficient for the reduction to utilize the fact that many terms in the IBP equations are zero. We call the $B(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6, \nu_7, \nu_8, \nu_9)$ as master-integrals.

Now we proceed to find algebraic equations for the integrals of the box topology. The easy way to derive such identities is the IBP method [15]; we multiply the integrand with a loop or external momentum and differentiate it with the loop momentum. These total derivatives integrate to zero. 12 IBP identities are obtained.

In summary, the algorithm requires a successive generation of identities with terms of increasing complexity. The newly added equations usually contain terms which are also found in equations generated at earlier stages; this produces small subsystems of coupled algebraic identities. A series of substitutions diagonalizes these algebraic subsystems and yields complicated integrals expressed in terms of master integrals. The algorithm is a clever implementation of Gauss elimination. It exploits the fact that Feynman integrals can be ordered according to a very simple criteria.

In the case of our Feynman diagram, after all the mentioned procedures, eventually we obtain 7 (seven) master-integrals to integrate.

It is possible and we intend to write a program, such as AIR, for parallel computing, which will be more efficient, as for obtaining these seven master-integrals in one core case 8 hours of computer calculations are required.

For calculating these seven master-integrals, we need to use the method described below.

2.4. Sector Decomposition

Sector decomposition is a constructive procedure to separate overlapping infrared divergences in multiloop integrals. Working with parametric representation in $d = 4 - 2\varepsilon$ dimensions, adequate subtractions lead to the Laurent series in ε, where the coefficients of the pole and finite terms are sums of regular parameter integrals that can be evaluated numerically. This algorithm fully automates by implementing it into algebraic manipulation programs and is

applied to calculate some nontrivial 2-loop 4-point and 3-loop 3-point Feynman diagrams numerically .

In perturbative QCD, the calculation of infrared safe quantities has to be organized in such a way that the infrared poles stemming from virtual and real higher order corrections cancel.

The IR singularity structure of higher-loop Feynman diagrams was investigated in four dimensions, and later in [18] in the context of dimensional regularization and factorization in QCD. Working in dimensional regularization, subtraction procedures are well known for UV poles, and also for IR divergences present in Euclidean space (see e.g. [18]). On the other hand, no general subtraction scheme for soft and collinear IR singularities arising in Minkowski space is known for individual graphs. The method we present in this paper has been designed to isolate poles in the dimensional regulator ε for an arbitrary Feynman graph. Although the method works also for one-loop integrals, its virtues show up rather in two– or higher loop integrals with N ≥ 3 external legs, at least one of them being massless. It allows one to disentangle the overlapping soft and collinear divergent regions in Feynman parameter space by dividing the latter into sectors where parameters can get singular only in an independent manner. Then, by adding and subtracting adequate counterterms, one can isolate the singular parts and perform the integrations over the corresponding parameters analytically. The remaining regular integrals are in general too complex for analytical integration, but they can be integrated numerically. This procedure is quite general and can in principle be applied to graphs with an arbitrary number of loops and legs, the limitations being only disk space and computing time.

By using sector decomposition method we obtain numerical results for all master-integrals.

There are the largest opportunities for the use of parallelizing in this section. It is possible to simultaneously compute every single master-integral, and certainly, at the same time calculate different sectors of each master-integral decomposed by the method.

2.5. Mellin-Barnes representation

To solve integrals obtained after Feynman parameterisation, which contain denominators of the form

$$\frac{1}{\left(m_c^2 P_1 + m_b^2 P_2\right)^{\alpha}}, \qquad (2.3)$$

where $P1$ and $P2$ are polynomials in the Feynman parameters, we need to apply Mellin–Barnes representation [19]

$$\frac{1}{(x+y)^{\alpha}} = \frac{1}{\Gamma(\alpha)} \int_{\gamma} \frac{ds}{2\pi i} \frac{x^s}{y^{\alpha+s}} \Gamma(\alpha+s)\Gamma(-s), \qquad (2.4)$$
$$x = m_c^2 P_1, \quad y = m_b^2 P_2$$

where the integration contour C runs from $-i\infty$ to $+i\infty$ in a way that it separates the poles generated by the two functions. This representation proves to be useful because then the integration over the Feynman parameters becomes trivial. Finally, we close the integration contour C by a half-circle with infinite radius at either of the sides and sum up the enclosed residues.

3. SUMMARY

In this paper we presented the numerical calculation methods for multiloop Feynman diagram integrals, described numerical methods which can be parallelized using multicore processors in more details. The description of these methods are shown on the example of $\overline{B} \to X_s\gamma$ decay at $O\left(\alpha_s^2\right)$.

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