An Upper Bound for the Parameter μ_{21} of Regular Graphs

Narine Davtyan
Ijevan Branch of Yerevan State University
Ijevan, Republic of Armenia
e-mail: nndavtyan@gmail.com

Rafayel Kamalian
The Institute for Informatics and Automation Problems
of NAS RA
Yerevan, Republic of Armenia
e-mail: rrkamalian@yahoo.com

ABSTRACT

An upper bound for the parameter μ_{21} of regular graphs is found.

Keywords

Regular graph, proper edge coloring, interval spectrum, game.

1. INTRODUCTION

We consider undirected connected simple finite graphs, which contain at least one edge. $V(G)$ and $E(G)$ denote the sets of vertices and edges of a graph G, respectively. An arbitrary nonempty finite subset of consecutive integers is called an interval. A function $\varphi : E(G) \to \{1, 2, ..., t\}$ is called a proper edge t-coloring of a graph G, if it satisfies the following two conditions: 1) for $\forall i$, $1 \leq i \leq t$, $\exists e^{(i)} \in E(G)$ with $\varphi(e^{(i)}) = i$, 2) for arbitrary adjacent edges $e' \in E(G)$ and $e'' \in E(G)$ $\varphi(e') \neq \varphi(e'')$. The least value of t, for which there exists a proper edge t-coloring of a graph G, is denoted by $\chi'(G)$. For a graph G and for $\forall t$, satisfying the inequality $\chi'(G) \leq t \leq |E(G)|$, we denote by $\alpha(G, t)$ the set of all proper edge t-colorings of G. Let us also define the set $\alpha(G)$ of all proper edge colorings of a graph G:

$$\alpha(G) \equiv \bigcup_{t=\chi'(G)}^{|E(G)|} \alpha(G, t).$$

The terms and concepts which are not defined here can be found in [1].

If G is a graph, $x \in V(G)$, $\varphi \in \alpha(G)$, then let us define the spectrum $S_G(x, \varphi)$ of the vertex x at the coloring φ as follows: $S_G(x, \varphi) \equiv \{\varphi(e)/e \in E(G), e \text{ is incident with } x\}$; let $f_G(\varphi) \equiv |\{x \in V(G)/\ S_G(x, \varphi) \text{ be an interval}\}|$.

If G is a graph and t is an integer satisfying the inequality $\chi'(G) \leq t \leq |E(G)|$, let us set [2]:

$$\mu_1(G, t) \equiv \min_{\varphi \in \alpha(G,t)} f_G(\varphi),$$

$$\mu_2(G, t) \equiv \max_{\varphi \in \alpha(G,t)} f_G(\varphi).$$

For any graph G, set [2]:

$$\mu_{11}(G) \equiv \min_{\chi'(G) \leq t \leq |E(G)|} \mu_1(G, t),$$

$$\mu_{12}(G) \equiv \max_{\chi'(G) \leq t \leq |E(G)|} \mu_1(G, t),$$

$$\mu_{21}(G) \equiv \min_{\chi'(G) \leq t \leq |E(G)|} \mu_2(G, t),$$

$$\mu_{22}(G) \equiv \max_{\chi'(G) \leq t \leq |E(G)|} \mu_2(G, t).$$

Clearly, the parameters μ_{11}, μ_{12}, μ_{21} and μ_{22} are correctly defined for an arbitrary graph.

Let us note that exact values of the parameters μ_{12} and μ_{21} have certain game interpretations. Suppose that all edges of a graph G are colored in the game of Alice and Bob with antagonistic interests and asymmetric distribution of roles. Alice determines the number t of colors in the future proper edge coloring φ of the graph G, satisfying the condition $\chi'(G) \leq t \leq |E(G)|$, Bob colors edges of G with t colors.

When Alice aspires to maximize, Bob aspires to minimize the value of the function $f_G(\varphi)$, and both players choose their best strategies, then at the finish of the game exactly $\mu_{12}(G)$ vertices of G will receive an interval spectrum.

When Alice aspires to minimize, Bob aspires to maximize the value of the function $f_G(\varphi)$, and both players choose their best strategies, then at the finish of the game exactly $\mu_{21}(G)$ vertices of G will receive an interval spectrum.

The exact values of the parameters μ_{11}, μ_{12}, μ_{21} and μ_{22} were found for simple paths, simple cycles and simple cycles with a chord [3, 4], "Möbius ladders" [2, 5], complete graphs [6], complete bipartite graphs [7, 8], prisms [9, 5], n-dimensional cubes [9, 10] and the Petersen graph [11]. The exact values of μ_{11} and μ_{22} for trees are found in [12]. The exact value of μ_{12} for an arbitrary tree is found in [13] (see also [14, 15]).

In this paper an upper bound for the parameter μ_{21} of regular graphs is found.

2. RESULT

Theorem 1. *If G is an arbitrary r-regular graph, $r \geq 2$, then*

$$\mu_{21}(G) \leq \left\lfloor \frac{r \cdot |V(G)| - 2}{2 \cdot (r - 1)} \right\rfloor.$$

REFERENCES

[1] D.B. West, "Introduction to Graph Theory", *Prentice-Hall*, New Jersey, 1996.

[2] N.N. Davtyan, R.R. Kamalian, *On boundaries of extremums of the number of vertices with an interval*

spectrum among the set of proper edge colorings of "Möbius ladders" with t colors under variation of t, Proc. of the 3^{th} Ann. Sci. Conf. (December 5–10, 2008) of the RAU, Yerevan, 2009, pp. 81–84 (in Russian).

[3] N.N. Davtyan, R.R. Kamalian, *On properties of the number of vertices with an interval spectrum in proper edge colorings of some graphs*, the Herald of the RAU, 2, Yerevan, 2009, pp. 33–42.

[4] N.N. Davtyan, R.R. Kamalian, *Some properties of the number of vertices with an interval spectrum in proper edge colorings of graphs*, The Collection "Akunq" of Scientific Papers of Ijevan Branch of Yerevan State University, Yerevan, 2012, pp. 18–27.

[5] N.N. Davtyan, A.M. Khachatryan, R.R. Kamalian, *On Boundaries of Extrema of the Number of Vertices with an Interval Spectrum on the Sets of Proper Edge t-colorings of Some Cubic Graphs under Variation of t*, International Mathematical Forum, Vol. 8, 2013, no. 24, pp. 1195 - 1198, http://dx.doi.org/10.12988/imf.2013.3491.

[6] A.M. Khachatryan, *On boundaries of extremums of the number of vertices with an interval spectrum among the set of proper edge colorings of complete graphs with t colors under variation of t*, Proc. of the 5^{th} Ann. Sci. Conf. (December 6–10, 2010) of the RAU, Yerevan, 2011, pp. 268–272 (in Russian).

[7] A.M. Khachatryan, *On the parameters μ_{11}, μ_{12} and μ_{22} of complete bipartite graphs*, the Herald of the RAU, 1, Yerevan, 2011, pp. 76–83 (in Russian).

[8] R.R. Kamalian, A.M. Khachatryan, *On the sharp value of the parameter μ_{21} of complete bipartite graphs*, the Herald of the RAU, 2, Yerevan, 2011, pp. 19–25 (in Russian).

[9] R.R. Kamalian, A.M. Khachatryan, *On properties of a number of vertices with an interval spectrum among the set of proper edge colorings of some regular graphs*, Proc. of the 6^{th} Ann. Sci. Conf. (December 5–9, 2011) of the RAU, Yerevan, 2012, pp. 62–65 (in Russian).

[10] A.M. Khachatryan, R.R. Kamalian, *On the μ-parameters of the graph of the n-dimensional cube*, Book of abstracts of the International Mathematical Conference on occasion to the 70th year anniversary of Professor Vladimir Kirichenko, June 13-19 (2012), Mykolaiv, Ukraine, pp. 38–39.

[11] N.N. Davtyan, *On the μ-parameters of the Petersen graph*, http://arxiv.org/abs/1307.2348

[12] N.N. Davtyan, *On the least and the greatest possible numbers of vertices with an interval spectrum on the set of proper edge colorings of a tree*, Math. Problems of Computer Science, Vol. 32, Yerevan, 2009, pp. 107–111 (in Russian).

[13] N.N. Davtyan, R.R. Kamalian, *On the parameter μ_{12} of a tree*, Proc. of the 4^{th} Ann. Sci. Conf. (November 30 – December 4, 2009) of the RAU, Yerevan, 2010, pp. 149–151 (in Russian).

[14] N.N. Davtyan, R.R. Kamalian, *On an algorithm of evaluation of the exact value of the parameter μ_{12} of an arbitrary tree*, the Herald of the RAU, 1, Yerevan, 2011, pp. 57–63 (in Russian).

[15] N.N. Davtyan, *On a property of the parameter μ_{12} of trees of special kind*, the Herald of the RAU, 2, Yerevan, 2010, pp. 77–82 (in Russian).