On the Length of the Minimal Testor for Some Class of Binary Tables

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ABSTRACT

Defines Testor and minimal Testor of binary tables and calculates the asymptotics of length of minimal Testor for some class of tables.

Keywords

Test, checking test, testor

1. Many problems of discrete mathematics, including problems which are traditionally considered to be complex, lead to the following tasks [1]. We have a table of zeros and ones (binary table), whose rows are distinct. We need to find minimal amount of rows, which make a subtable, the lines of which are also different from one to another (minimal test), or the minimal amount of rows, the lines of which are different from the first one (minimal checking test). For example, those can be tasks of controlling electrical systems, tasks of controlling functional schemes, tasks of minimal covering sets of their subsets, any problems of graph theory, etc. A lot of work, for example, [3] is devoted to research length of the minimal test for "almost all" of tables (see definitions below). In [4] for a class table it shows, that "almost all" of the table are the minimal checking tests no more than two lengths. In this paper we address similar question for Testor of binary tables, which are a generalization of checking test.

2. Give a necessary definitions.

Let $\{M(n)\}_{n=1}^{\infty}$ be the collection of sets, such that $|M(n)| \to \infty$ when $n \to \infty$, ($|M|$ is the power of the set M), and $M^s(n)$ be the subset of the all elements from $M(n)$, which have the any property S. We say, that almost all the elements of the set $M(n)$ have the property S, if $|M^S(n)|/|M(n)| \to 1$, when $n \to \infty$.

We denote by T_{mn} the set of all the binary tables, composed of m rows and n columns, whose rows are distinct. Subset of rows of table $T \in T_{mn}$, which make subtable, the subrows of which are different from the first one, is called a checking test. The number of columns in the checking test is called its length. Minimal checking tests of the table is called the checking test, which has the minimum length.

Consider the generalization of checking test. We fix the first $1 < p < m$ row of the table $T \in T_{mn}$. The subset of columns in a table T, that defines the subtable, where all of the first p subrows are different from the rest $m-p$ subrows, is called p - Testor, or just Testor of this table. The number of columns in the testor is called its length. Minimal testor of the table is called the testor, having the minimum length.This definition is essentially the same as the concept of Testor, which introduced Yu. I. Zhuravlev in [2]. When $p=1$ we get a checking test of table.

Everywhere under the $\log$ refers to the logarithm to the base 2. Parameter $n \to \infty$. If wherein $m \to \infty$, it is assumed, that $m^2/2^n \to 0$, $p/m \to 0$ or $p/m \to 1$, and this assumption is not indicated in the statements.

3. Following statements occur:

Theorem1. If $n \geq k \geq \log(p(m-p)) + \varphi(n)$, where $\varphi(n) \to \infty$ when $n \to \infty$ arbitrarily slowly, then for almost all tables $T \in T_{mn}$ any subset of k columns of the table is its Testor.

We denote by $\mu(T)$ the length of the minimal Testor of table $T \in T_{mn}$.

Corollary from Theorem1: For almost all the table $T \in T_{mn}$ for the length of the minimal Testor occurs $\mu(T) \leq [\log(p(m-p))] + 1$

Theorem 2. If $p(m-p) > \ln n$, then for almost all the tables $T \in T_{mn}$ for the length of the minimal Testor occurs $\mu(T) \geq [\log(p(m-p)) - \log\log(p(m-p)) - \log\ln n]$

From these theorems the following statement results:

Theorem 3. If $p(m-p) > \ln n$, then for almost all the tables $T \in T_{mn}$ for the length of the minimal Testor $\mu(T)$ occurs $\mu(T) \sim \log(p(m-p))$.

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