Semigroup Models of Cyber-physical Systems

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ABSTRACT

A new mathematical model for cyber-physical systems based on algebraic theory of interaction and the technology of insertion modeling is considered. It is based on the notion of a semigroup transition system and is abstract enough to describe a wide class of timed transition systems. Multi agent cyber-physical systems are formalized on a base of the theory of interaction of agents and environments. Using this new model allows efficient algorithms to analyze, synthesize, and verify models of cyber-physical systems.

Keywords

Transition system, semigroup, trace equivalence, hybrid automaton.

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A *semigroup transition system* consists of three components $< S, G, H >$ where S represents the set of states, G represents a parameterized transition relation, and H represents a semigroup of traces (observed processes of semigroup transition system). In contrast to the labeled transition system, the transition relation is parameterized by the time parameters

$$G = \{g_t \subseteq S \times H \times S | t \in T \subseteq R\}.$$

Here T represents a set of time moments, which is assumed to be additive subsemigroup of the semigroup of real numbers R under addition.

We use the following notation:

$$s \xrightarrow{h}_t s' \quad \Leftrightarrow \quad (s, h, s') \in g_t,$$
$$s \xrightarrow{h} s' \quad \Leftrightarrow \quad \exists\, t(s, h, s') \in g_t.$$

The components of the semigroup transition system should satisfy the following axioms:

A1. $t, t' \in T \Rightarrow t + t' \in T$,

A2. $\forall\, s\, \exists\, (h, s')(s \xrightarrow{h} s')$,

A3. $s \xrightarrow{h}_t s' \xrightarrow{h'}_{t'} s'' \Rightarrow s \xrightarrow{hh'}_{t+t'} s''$,

A4. $s \xrightarrow{h}_{t+t'} s', t, t' \in T$
$\Leftrightarrow \exists(h', h'', s'')(h = h'h'', s \xrightarrow{h'}_t s'' \xrightarrow{h''}_{t'} s')$.

The main difference of semigroup transition system from labeled transition systems is that the labels of transitions are not actions but traces. It allows to use as a trace semigroup not only a free semigroup but also semigroups which have no unreducible sets of generators (for example the set of reals or functions of continuous time defined on finite semiopened intervals).

The axiom A3 (folding axiom) together with the axiom A4 (unfolding axiom) allows to represent arbitrary finite history as a sequence of transitions of arbitrary durations.

The notion of semigroup transition system generalizes the labeled transition systems as well as timed and hybrid automata. Especially it was proved that Henzinger hybrid automaton can be equivalently represented by means of semigroup transition system.

A semigroup transition system can be modeled by labeled transition system if we change atransition $s \xrightarrow{h}_t s'$ by $s \xrightarrow{(h,t)} s'$ with the set of actions $H \times T$. It is a discrete transition system and all notions of an algebraic theory of interactions such that trace equivalence and bisimilarity, behavior algebra, symbolic modeling, and predicate transformers are defined for these kind of systems, and can be successfully transferred to semigroup transition systems.

The trace equivalence of states is defined using the semigroup product. Let

$$s_0 \xrightarrow{h_1}_{t_1} s_1 \xrightarrow{h_2}_{t_2} s_2 \cdots s_{n-1} \xrightarrow{h_n}_{t_n} s_n$$

be a finite history. Then a sequence $(h_1, h_2, , h_n)$ as usually is called a *trace* and a pair $(h_1 h_2 \cdots h_n, t_1 + t_2 + \cdots + t_n)$ is called a *normalized trace* generated by this history. Let $L(s)$ be a set of normalized traces generated by histories which start from the state s. The states s and s' are called transition equivalent if $L(s) = L(s')$.

Cyber-physical systems are modeled by semigroup attribute environments with semigroup agents inserted into these environments. The main difference between semigroup attribute environments and ordinary attribute environments is the following. All attributes are divided to *discrete* and *continuous* ones. Continuous attributes change their values continuously according to their evolution low which can be changed by discrete control systems. Discrete attributes change their values only at the moments when corresponding actions are performed. At other times, they retain the values obtained at the last change.

The virtual insertion machine has been designed for the analyses, verification, generating tests, and performing optimization transformations of models of cyber-physical systems.

REFERENCES

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