On the Structre of Positive and Strongly Positive Arithmetical Sets

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ABSTRACT

The relations between the classes of positive and strongly positive arithmetical sets are investigated. Some simplified form for the logical representation of the class of positive sets is obtained. It is proved that the logical representation of the class of positive sets is obtained when the list of operations $\&, \vee$ in the definition of the class of strongly positive sets is replaced by the list $\exists, \&, \vee$.

Keywords

Positive set, strongly positive set, arithmetical set, signature.

The notions of positive set and strongly positive set are defined and investigated in ([1]-[3]).

By N we denote the set of non-negative integers, $N = \{0, 1, 2, ...\}$. An n-dimensional arithmetical set is defined as a set of n-tuples $(x_1, x_2, ..., x_n)$, where $x_i \in N$ for $1 \le i \le n$. The notion of arithmetical formula in the signature $(0, =, S)$, where $S(x) = x + 1$, is defined as in ([1]-[3]). Let us recall the definitions of positive and strongly positive sets. An arithmetical set is said to be positive (see [1]) if it can be defined by an arithmetical formula in the signature $(0, =, S)$ containing only logical operations $\exists, \&, \vee, \neg$ and such that any symbol of negation $\neg$ in this formula relates to an elementary formula containing no more than one variable. An arithmetical set is said to be strongly positive (see [2]-[3]) if it can be defined by an arithmetical formula in the signature $(0, =, S)$ containing only logical operations $\&, \vee$ and the elementary formulas having one of the forms $x = a$ (where $a \in N$), $x = y$, $x = S(y)$, $\neg(x = 0)$ (where x and y are variables).

Theorem. An arithmetical set is positive if and only if it can be defined by an arithmetical formula containing only the logical operations $\exists, \&, \vee$ and the elementary formulas of the form $x = 0$ or $x = S(y)$, where x and y are variables.

Corollary. The logical description of the class of positive sets is obtained from the logical description of the class of strongly positive sets by replacing the list of operations $\&, \vee$ by the list $\exists, \&, \vee$.

ACKNOWLEDGEMENT

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