Volterra Polynomials in the Intelligent Control System of Wind Power Plant Operation

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ABSTRACT
The paper is focused on the possibility of integrating situation control and simulation modeling based on the Volterra integro-power series when describing transient processes in a nonlinear dynamic object of input-output type. A reference object was represented by a model of horizontal-axis wind turbine. The research deals with nonlinear dynamics of a rotation speed of wind turbine components depending on the blade angle and wind speed. A two-level technology is applied to prevent emergency situations. It consists of a quantitative analysis and a qualitative analysis. The first level is implemented using the situation analysis tools based on semantic modeling. At the second level the simulation modeling is performed. An algorithm for construction of a set of current situations in the semantic model is based on the Volterra polynomial integral equations. The Newton-Kantorovich method was applied to develop an iterative method to approximately solve respective nonlinear equations. The paper demonstrates the specific features of numerically solving the studied equations with a fixed length of mantissa in machine representation of a floating-point number.

Keywords
nonlinear dynamic systems, Volterra polynomials, iteration algorithm, numerical solution, MAPLE.

1. INTRODUCTION
Evolution of the conception of intelligent power systems, which is called Smart Grid in the other countries, requires a principally new approach to the monitoring and control of such objects [1]. Melentiev Energy Systems Institute has developed a two-level technology for the creation of intelligent tools to comprehensively investigate power facilities. This technology, including the methods of situation analysis [2, 3], is aimed at preventing emergency situations [4]. Current state and trends in the development of situation control are described in [5]. Traditionally, the situation analysis, when used to study the energy security problems, is made on the basis of semantic models [6].These models represent one of the advanced artificial intelligence areas and are related to the description of a subject domain with the help of basic notions and relations among these notions. The cause-effect relations are modeled normally by the ontological, cognitive and eventual modeling tools which when applied to the description of nonlinear dynamics of technical (energy) objects require the tools of mathematical modeling [7]. A brief analysis of modern methods for the identification of nonlinear systems, including advantages and disadvantages of neural networks, genetic algorithms, algorithms of self-organization as well as the algorithms based on the functional series of Viner and Volterra is presented in [8]. According to [8, 9] the Volterra integro-power series are the most preferable as they can be applied to different operating conditions of the studied object and can interact with a wide range of technical objects of "input-output" type.

To construct the non-stationary integral model

$$\sum_{p=1}^{N} \sum_{1\le i_1\le \ldots \le i_p \le 2} \phi_{i_1\ldots i_p}(t) = y(t), \qquad (1)$$

$$\phi_{i_1\ldots i_p}(t) = \int_0^t \ldots \int_0^t K_{i_1\ldots i_p}(t, s_1, ..., s_p) \prod_{j=1}^{p} x_{i_j}(s_j) ds_j,$$

where $t \in [0, T]$, $y(0) = 0$, it is necessary to recover the Volterra kernels $K_{i_1\ldots i_p}$ symmetric with respect to variables $s_1, \ldots, s_p$, which are called multidimensional transfer functions [10]. The non-stationarity is understood in the sense that Volterra kernels in (1) vary with time. Currently a lot of methods are developed to determine the dynamic characteristics $K_{i_1\ldots i_p}$ in both frequency domain and time domain. Their practical application is most often complicated by an extremely large amount of calculations. Therefore, in their studies in this area researchers normally seek to simplify the techniques [11, 12].

The most widely spread methods for solving the problem of the Volterra kernel identification in the time space are based on specifying pulse and staged input signals [13]. Many physical processes, however, do not allow pulse inputs [14]. In the case of multi-staged signals in the form of a combination of Heaviside functions, the decomposition of the system output into constituents is normally performed by the method of least squares [15, 16] and neural networks [17, 18].

The technique for the identification of $K_{i_1\ldots i_p}$ [19], developed by the authors of this paper, is based on specifying an $(p + 1)$-parametric family of piecewise constant inputs in the form of a combination of Heaviside functions with deviating argument. In this case the initial problem is reduced to solving linear integral multidimensional Volterra equations of the first kind with varying upper and lower limits. Corresponding integral equations have explicit inverse formulas.

Further we assume that the task of identification of kernels $K_{i_1 \dots i_p}$ in (1) is somehow solved. The goal of this paper is to consider specific features of building a system of situation control of nonlinear dynamic objects on the basis of Volterra polynomials, develop a numerical method for solving the Volterra polynomial equations and demonstrate its implementation.

2. DESCRIPTION OF THE SUBJECT DOMAIN

Consider a horizontal axis wind turbine plant controlled with respect to blade lean angle. The plant is represented using the techniques [20, 21]:

$$\frac{d\omega_T}{dt} = \frac{M_{WT}(t) - M_{CG}(t)}{J}, \ \omega_T(0) = 0, \qquad (2)$$

$$z(t) = \left(\frac{1}{Z(t) + 0.08b(t)} - \frac{0.035}{b^3(t) + 1} \right)^{-1},$$

$$C_p(t) = 0.22 \left(\frac{116}{z(t)} - 0.4b(t) + 5 \right) \exp \left(-\frac{12.5}{z(t)} \right),$$

$$Z(t) = \frac{\omega_T(t) R}{V(t)}, \ M_{WT}(t) = \frac{\rho S C_p(t) V^3(t)}{2\omega_T(t)},$$

where $\omega_T(rad/s)$ — rotational speed of wind turbine elements, $M_{WT}(N \cdot m)$ — created by aerodynamic force, $M_{CG}(N \cdot m)$ — load resistance torque, $J(kg \cdot m^2)$ — moment of inertia of the wind turbine rotating parts, $\rho(kg \cdot m^2)$ — air density, $S(m^2)$ — blade-swept area, $R(m)$ — wind wheel radius, $b(deg)$ – blade lean angle, $V(m/s)$ — wind speed; dimensionless magnitudes: C_p — wind energy efficiency, Z — speed, z — current value of speed. The difference approximation of the output $\Delta\omega_T(\tau) = \omega_T(\tau) - \omega_{T_0}$ — of system (2) to the inputs $\Delta b(t) = b(t) - b_0$ and $\Delta V(t) = V(t) - V_0$ is implemented using the 4-th order Runge–Kutta method in the MatLab environment. A flow chart of the control object is presented in Fig. 1.

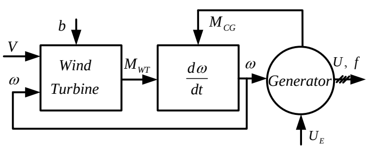

Figure 1. Flow chart for the wind turbine plant.

An ontological model of the wind turbine situation control was developed under the assumption that there are no external disturbances except for the relations of input-output type. The rules of representation of data and ranges of variation in the parameters were fixed with respect to some chosen steady state of the modeled system. A scheme and description of order of the dynamic object output calculation is presented in [22]. Software package "Dinamika" [23] was developed to identify and model the reference power object (2). We will study a problem of automatic control, related to the search of the input $x_1(t) \equiv \Delta b(t)$, which at set input $x_2(t) \equiv \Delta V(t)$ maintains the output $\Delta\omega_T(t) = y(t)$ at a desired level y^* (indices "0" are used to denote the parameters of the initial conditions). To suppress overshoot and provide online control of transient processes we introduce a trained controller (in terms of [24]) on the basis of the Volterra polynomials. It should be noted that in [24] an example of neural network was presented as a trained controller. The application of Volterra polynomials for these purposes does not contradict the recommendations in [24]. In [25] the authors show that some classes of neural networks are equivalent to the final section of the Volterra series.

Mathematically, the problem of search for the control action $x_1(t)$ can be reduced to Volterra polynomial integral equation (1). The papers [26, 27] are devoted to the theory and application of quadrature methods (of right and middle rectangles) to numerically solve (1) at $N = 2, 3$ in case of scalar input signal. The main feature of the polynomial equation (1) at $N > 1$ lies in the fact that its (unique) continuous solution is of a local character, so that the magnitude T^*, should be sufficiently small. In the event that at some inputs $x_2(t)$ the interval of the existence of sought solution $x_1(t)$ is smaller than the time of transient process, this result is interpreted as a potential loss of controllability of the investigated process, and, consequently, an emergency. Thus, the formation of a set of current situations in the semantic model implies tracing the scenarios of system response to the input with the help of a simulation model.

3. NUMERICAL SOLUTION OF VOLTERRA EQUATION

Consider numerical solution of (1) for the case of the vector signal $x(t) = (x_1(t), x_2(t))^T$ with the help of the Newton–Kantorovich method [28]. In practice the problem is usually limited to $N \leq 3$, and the current section addresses exactly the equations of the second and third degree. To provide a better understanding of (1), it would be useful to consider a test equation with constant kernels: $K_{i_1 \dots i_p}(t, s_1, \dots, s_p) = K_{i_1 \dots i_p}$. Herein all function and functional spaced are assumed to be real. The change $\theta_i(t) = \int_0^t x_i(s) ds$ reduces the original problem to finding the continuous solution $\Theta^*(t) = \theta_1^*(t)$ to the equations of the second $(N = 2)$

$$f(\Theta(t)) \equiv \sum_{i=1}^{2} K_i \theta_i(t) + \sum_{i=1}^{2} \sum_{j=1}^{i} K_{ji} \theta_i(t) \theta_j(t) = y(t) \qquad (3)$$

and the third degree $(N = 3)$

$$f(\Theta(t)) \equiv \sum_{i=1}^{2} K_i \theta_i(t) + \sum_{i=1}^{2} \sum_{j=1}^{i} K_{ji} \theta_i(t) \theta_j(t) + \qquad (4)$$

$$+ \sum_{i=1}^{2} \sum_{j=1}^{i} \sum_{k=1}^{j} K_{kji} \theta_i(t) \theta_j(t) \theta_k(t) = y(t),$$

where $y(0) = 0$, $y'(t) \in C_{[0,T]}$, $K_1 \neq 0$, $T < T^*$. Consider

$$P(\Theta(t)) \equiv f(\Theta(t)) - y(t). \qquad (5)$$

The iterative process for solving (5) by the Newton–Kantorovich method has the form:

$$\Theta_m = \Theta_{m-1} - \left[P'(\Theta_{m-1}) \right]^{-1} (P(\Theta_{m-1})), m = 1, 2, ..., \qquad (6)$$

where

$$P'(\Theta_{m-1}) = K_1 + K_{12}\theta_2(t) + 2K_{11}\Theta_{m-1}(t), \qquad (7)$$

if $f(\Theta(t))$ is defined by (3), and

$$P'(\Theta_{m-1}) = K_1 + K_{12}\theta_2(t) + K_{122}\theta_2^2(t) + \qquad (8)$$

$$+ 2(K_{11} + K_{112}\theta_2(t))\Theta_{m-1}(t) + 3K_{111}\Theta_{m-1}^2(t),$$

if $f(\Theta(t))$ is defined by (4). Taking into account (6) and (7), the approximation $\Theta_m(t)$ is defined by the formulas

$$\Theta_m(t) = \frac{\Phi_{m-1}(t)}{K_1 + K_{12}\theta_2(t) + 2K_{11}\Theta_{m-1}(t)}, \qquad (9)$$

$$\Phi_{m-1}(t) = K_{11}\Theta_{m-1}^2(t) - K_2\theta_2(t) - K_{22}\theta_2^2(t) + y(t),$$

where

$$\Theta_0(t) = \frac{y(t) - K_2\theta_2(t) - K_{22}\theta_2^2(t)}{K_1 + K_{12}\theta_2(t)}$$

— is the initial approximation. Similarly, it follows from (6) and (8) that

$$\Theta_m(t) = \qquad (10)$$

$$= \frac{\Psi_{m-1}(t)}{a(t) + 2\left(K_{11} + K_{112}\theta_2(t)\right)\Theta_{m-1}(t) + 3K_{111}\Theta_{m-1}^2(t)},$$

$$a(t) = K_1 + K_{12}\theta_2(t) + K_{122}\theta_2^2(t),$$

$$\Psi_{m-1}(t) = 2K_{111}\Theta_{m-1}^3(t) + \left(K_{11} + K_{112}\theta_2(t)\right)\Theta_{m-1}^2(t) -$$

$$- K_2\theta_2(t) - K_{22}\theta_2^2(t) - K_{222}\theta_2^3(t) + y(t),$$

$$\Theta_0(t) = \frac{y(t) - K_2\theta_2(t) - K_{22}\theta_2^2(t) - K_{222}\theta_2^3(t)}{K_1 + K_{12}\theta_2(t) + K_{122}\theta_2^2(t)}.$$

Remark. Application of the modified Newton–Kantorovich method to Eqs. (3) and (4) derives the following computational formulas

$$\hat{\Theta}_m(t) = \frac{\hat{\Phi}_{m-1}(t)}{K_1 + K_{12}\theta_2(t) + 2K_{11}\Theta_0(t)}, \qquad (11)$$

$$\hat{\Phi}_{m-1}(t) = 2K_{11}\hat{\Theta}_{m-1}(t)\Theta_0(t) - K_{11}\hat{\Theta}_{m-1}^2(t) -$$

$$- K_2\theta_2(t) - K_{22}\theta_2^2(t) + y(t)$$

and

$$\hat{\Theta}_m(t) = \qquad (12)$$

$$= \frac{\hat{\Psi}_{m-1}(t)}{a(t) + 2(K_{11} + K_{112}\theta_2(t))\Theta_0(t) + 3K_{111}\Theta_0^2(t)},$$

$$\hat{\Psi}_{m-1}(t) = 2K_{112}\theta_2(t)\hat{\Theta}_{m-1}(t)\Theta_0(t) +$$

$$+ 3K_{111}\hat{\Theta}_{m-1}(t)\Theta_0^2(t) - K_{112}\theta_2(t)\hat{\Theta}_{m-1}^2(t) -$$

$$- K_{111}\hat{\Theta}_{m-1}^3(t) - K_{222}\theta_2^3(t) + \hat{\Phi}_{m-1}(t)$$

correspondingly.

Example 1. Let in (3) $K_1 = 1$, $K_2 = K_{11} = K_{22} = K_{12} = -1$, $\theta_2(t) = t$, $y(t) = t$. Then $\Theta_0(t) = \frac{2t+t^2}{1-t}$, $\Theta^*(t) = \frac{1-t-\sqrt{-3t^2-10t+1}}{2}$, $t \in [0, T^*)$, $T^* = \frac{2\sqrt{7}-5}{3}$.

Table 1. Numerical results for Example 1

m	$\Theta_m(t_k)$	$\|\|\varepsilon_{t_k}\|\|$	$\hat{\Theta}_m(t_k)$	$\|\|\hat{\varepsilon}_{t_k}\|\|$
1	0.2927	$0.2469 \cdot 10^{-1}$	0.2927	0.0247
2	0.3156	$0.1878 \cdot 10^{-2}$	0.3076	0.0098
3	0.3174	$0.1265 \cdot 10^{-4}$	0.3133	0.0042
4	0.3174	$0.5818 \cdot 10^{-11}$	0.3156	0.0018

Table 1 presents the results obtained by formulas (9) and (11) at $t_k = 0.09$, where m – the number iterations,

$$||\varepsilon_{t_k}|| = \max_{0 \le t \le t_k} |\Theta_m(t) - \Theta^*(t)|,$$

$$||\hat{\varepsilon}_{t_k}|| = \max_{0 \le t \le t_k} |\hat{\Theta}_m(t) - \Theta^*(t)|.$$

Example 2. Let in (4) $\theta_2(t) = t$, $K_1 = 1$, $K_2 = K_{11} = K_{22} = K_{12} = K_{111} = K_{112} = K_{122} = K_{222} = -1$, $y(t) = -t - \frac{t^2}{2} - \frac{3t^3}{2} - \frac{3t^4}{4} - \frac{t^5}{4} - \frac{t^6}{8}$, $t \in [0, 0.4]$. Then $\Theta^*(t) = \frac{t^2}{2}$. Let $\tilde{\Theta}_0(t) = \frac{t^2\left(-4+12t+6t^2+2t^3+t^4\right)}{8\cdot\left(t^2+t-1\right)}$ be the initial approximation, so that $\Theta_0(t) - \tilde{\Theta}_0(t) = \frac{t^3}{1-t-t^2}$. Table 2 presents the results obtained by formulas (10) and (12) at $t_k = 0.35$ and $\tilde{\Theta}_0(t)$.

Table 2. Numerical results for Example 2

m	$\Theta_m(t_k)$	$\|\|\varepsilon_{t_k}\|\|$	$\hat{\Theta}_m(t_k)$	$\|\|\hat{\varepsilon}_{t_k}\|\|$
1	$0.4266 \cdot 10^{-1}$	$0.1859 \cdot 10^{-1}$	0.0427	0.0186
2	$0.5998 \cdot 10^{-1}$	$0.1272 \cdot 10^{-2}$	0.0543	0.0069
3	$0.6124 \cdot 10^{-1}$	$0.6978 \cdot 10^{-5}$	0.0584	0.0028
4	$0.6125 \cdot 10^{-1}$	$0.1285 \cdot 10^{-11}$	0.0601	0.0012

Results of the computations show that the iterative algorithm with the chosen initial approximations converges to the exact value. The algorithm was implemented in MAPLE. We will include parameter φ in the generally accepted representation of the real number. The parameter is equal to the number of valid digits in the mantissa (starting from the left). Assume that the real number $x = s \cdot M \cdot 10^{-L+\rho}$ is specified by the set (s, M, ρ, φ), where $s \in \{-1, 0, +1\}$ is the sign of the number, $M \in \{10^{L-1}, 10^{L-1}+1, ..., 10^L - 1\} \cup \{0\}$ is mantissa of the number, L is number of mantissa positions, ρ is exponent part of the number. Table 3 presents the parameters $(1, M, -1, \varphi)$, which define $x \equiv \Theta_m(t_k)$ from Example 2.

Table 3. The relationship between L and φ

m	L	M	φ
2	7	5930645	2
	8	59969900	3
	9	599754850	4
3	17	60897699814714115	1
	18	611887532071769490	2
	19	6124215591066970715	4

In the course of the computational experiments it was revealed that in the neighborhood of the blow up point (T^*) we can observe a momentary loss of accuracy in the calculations with a fixed length of mantissa.

4. CONCLUSUONS

This work continues investigations started in [22]. On the basis of the quadratic Volterra polynomials we performed modeling of the control system for a dynamic object represented as a mathematical model of the wind generation plant with a horizontal rotation axis. We considered the specifics of creating situational control systems for nonlinear dynamic objects on the basis of the Volterra polynomials and developed a numerical method for solving polynomial Volterra equations. The implementation of the method in MAPLE showed that we have to take into consideration error generation mechanisms when performing computations.

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