Computer Simulation of Blood Flow in Arteries With Deformable Walls

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ABSTRACT
For the computer simulation of pulsating blood flow in arteries the blood was considered as a non-Newtonian liquid. The pulsating flow is created by the pulsating nature of the input boundary condition. The vessels are considered as thick-walled cylinders with deformable hyperelastic walls. The interaction of blood and vessel is defined by means of semi-slip boundary condition. Computer simulation was performed in software complex ANSYS with the use of the direct conjugating modul CFX and the module ANSYS "Multiphysics". As a result, the pressure and stress wave propagations in the vessel wall was obtained.

Keywords
Computer simulation, blood flow, deformable vessel wall, ANSYS, splitting method.

1. INTRODUCTION
One of the main difficulties, arising in the computer simulation of haemodynamics, lies in the complicated blood structure and its mechanical properties. As is known, the blood consists of liquid plasma and deformable blood cells, which range up to 46 % of the blood volume (Fig.1), and their mechanical properties and inner structure are have poorly been studied.

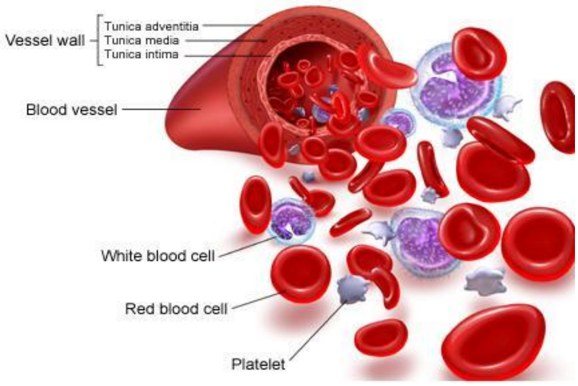

Fig. 1. The blood cells

The main part of blood cells (about 90 %) comprises erythrocytes. In blood flow they can create aggregates of column form, which also changes the flow pattern and its mechanical properties [1] (Fig. 2).

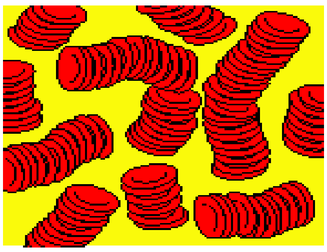

Fig. 2. Formation of the erythrocyte aggregates

In essence, the blood is a suspension, having too complicated and varied structure. In connection with this, the consideration of blood as a two-phase medium contains significant simplifications [2, 3]. For this reason we considered the blood as a liquid, but only if the blood vessel diameter is much greater than the dimensions of cells and their aggregates. Furthermore, since the presence of blood cells significantly influences its rheology and, as a consequence, affects the nature of the flow [4], blood may be considered only as a non-Newtonian liquid that, in its turn, affects the nature of the flow [4]. The special attention was paid to this effect in [5]. It is also important to take into account the non-Newtonian blood properties in cases of vessel pathologies [6]. Effects of the non-Newtonian viscosity of blood on flow in diseased arterial vessels even for steady flows was underlined in [7].

Besides, due attention should be given to the pulse nature of blood flow in arteries, which is shown in Fig. 3.

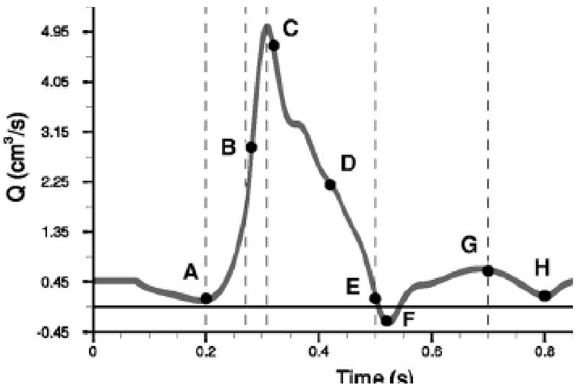

Fig. 3. The blood flow at the input cross-section

This figure was obtained during medical investigation of the patient. But as the vessel is further and further from the heart the pulse wave weakens and changes its form, and this circumstance should be taken into account when using this characteristic is used as a boundary condition.

Another difficulty is the complicated vessel structure and insufficient knowledge of vessel mechanical properties, which are essential for their modelling and the subsequent formulation of the "fluid-structure" problem as hydroelasticity one. As a result, the authors are forced to use only simplified models. In this connection it should be emphasized that this vessel property manifests in various degree for different vessels. Since the aorta is the most rigid vessel, its wall can be considered as undeformable [8-10]. However, vessels are located further from ramble walls and this should be taken into consideration (see, for example [11]).

Of great importance is the interaction between the blood and the vessel wall. It should be noted that the "no-slip" condition, which is usually used as a boundary condition when solving problems of viscous flow, is unacceptable for the adequate description of interaction between the blood and the vessel wall [12]. The matter is that the blood cells do not stick to a healthy vessel wall and slide along it due to their electrochemical properties. In this connection it would be appropriate to use the so-called “semi-slip” condition. The expediency of this condition application for smooth surfaces was pointed out in [13].

2. MATHEMATICAL MODELS

The proposed computer simulation is based on the use of mathematical models of blood and vessels.

2.1. Mathematical model of blood

The blood was considered as a non-Newtonian liquid with Power law for correlation between the stress tensor and the strain velocity tensor. This correlation can be written in the tensor form

$$\mathbf{T} = 2k\left|I_2\right|^{\left(\frac{n-1}{2}\right)}\mathbf{D}\,, \tag{1}$$

where **T** is the viscous stress tensor, **D** is the strain velocity tensor and I_2 is its second principal invariant of **D**. This law describes the properties not only of liquid but also the suspension, which is essentially the blood. In this case the governing equation can be written as follows:

$$\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \nabla\cdot(-p\mathbf{I}+\mathbf{T})\,, \tag{2}$$

where **v** is the blood velocity vector, **I** – the identity tensor. In addition to this the equation of continuity should be written

$$\operatorname{div}\mathbf{v} = 0 \tag{3}$$

The rest state was chosen as the initial conditions for the equations (1) and (2) with relationship (3), that is

$$\mathbf{v}\big|_{t=0} = 0 \tag{4}$$

After the calculation process starts, it continues until reaching a stationary regime.

The boundary conditions were set in the following manner. At the input cross-section of the considered vessel part the pulsating blood flow was given as it is shown in Fig. 2. The output cross-section is mentally placed so far that the velocity is constant and has only the longitudinal component *v*. The pressure is also constant, that is

$$v\big|_{output} = \text{const}\,,\ p\big|_{output} = \text{const} \tag{6}$$

As it is pointed above, the blood cells do not adhere to the vessel wall if its inner layer has no injury. That is why it was called to use the so called semi-slip condition was used as the boundary condition. In this case the longitudinal component of velocity decreases, to a certain preassigned value v_0 (Fig. 4), which is the boundary condition at the wall:

$$v(r_0,\varphi,t) = v_0 \tag{7}$$

The degree of semi-slip is defined by the parameter *b*.

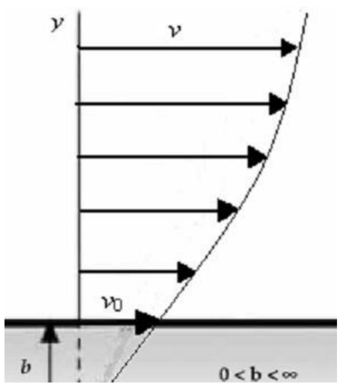

Fig. 4. The velocity profile for semi-slip condition

2.2. Mathematical model of blood vessels

Blood vessels are considered as thick-walled cylinders. The wall material is hyperelastic orthotropic and incompressible. In this case the wall mechanical properties are described by the specific deformation energy *W*. The equations of the vessel wall motion can be written in the following form:

$$\rho\frac{\partial^2\mathbf{u}}{\partial t^2} - \nabla\cdot\left(\mathbf{F}(u)\mathbf{S}(u)\right) = 0$$
$$\det\mathbf{F}(u) = 1$$
$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F}-\mathbf{I}\right),\ \ \mathbf{S} = \frac{\partial W(\mathbf{E})}{\partial\mathbf{E}},$$

where **u** – the wall displacement vector; *ρ*- the density of the wall material; **F** – the deformation gradient; **S** – the second Piola-Kirchhoff stress tensor and **E** – the Green strain tensor. The boundary conditions which connect the movements of the blood and the wall with regard to semi-slip condition are of the form:

$$\begin{aligned}
v\big|_{wall}\cdot\mathbf{n} &= -b\left.\left(\frac{\partial u}{\partial t}\right)\right|_{wall}\cdot\mathbf{n}\\
v\big|_{wall}\cdot\boldsymbol{\tau} &= \left.\left(\frac{\partial u}{\partial t}\right)\right|_{wall}\cdot\boldsymbol{\tau}\\
\sigma\big|_{wall}\cdot\mathbf{n} &= -\left.\left(\mathrm{F}(u)\mathrm{S}(u)\right)\right|_{wall}\cdot\mathbf{n}\\
\sigma\big|_{wall}\cdot\boldsymbol{\tau} &= -\left.\left(\mathrm{F}(u)\mathrm{S}(u)\right)\right|_{wall}\cdot\boldsymbol{\tau}
\end{aligned},$$

where **n** and **τ** are the normal and tangent vectors of boundary, $v\big|_{wall}$ and $\sigma\big|_{wall}$ are the blood velocity and stress on the wall. The stress normal component at the external wall boundary was taken to be zero.

3. CALCULATIONS

Calculations were performed in software complex ANSYS with the use of the direct conjugating of the program modul CFX, destined for the liquid flow simulation, and the module ANSYS “Multiphysics”, destined for simulation of the stress-strain state. For this purpose the three-space model of a blood vessel was constructed with help computer program CAD and then it was passed into ANSYS ICEM CFD for the of the construction of the three-space hexagonal deformable mesh with concentration near the wall. Exchange of the information between modules CFX and “Multiphysics” takes place at each time step, as it is shown in Fig. 5.

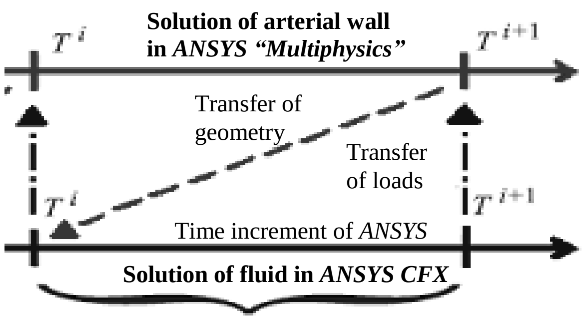

Fig. 5. The diagram of interaction between modules ANSYS CFX and ANSYS “Multiphysics”.

4. RESULTS

As results, the pressure wave propagation in the vessel and stress wave propagation in vessel wall together with the vessel strain during the pulse wave passage were obtained. The examples are shown in Fig. 6 and Fig. 7.

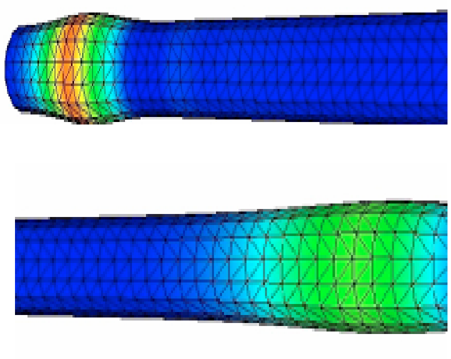

Fig. 6. The pressure wave propagation in the vessel.

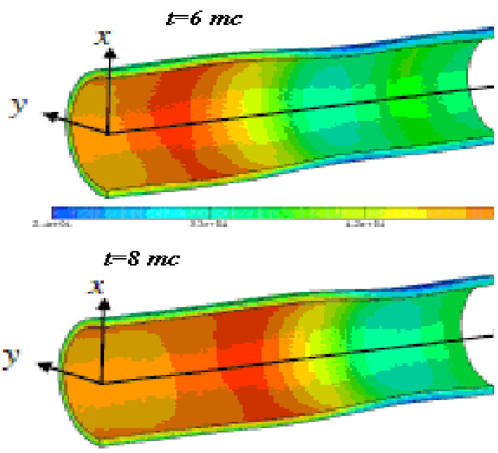

Fig. 7. The stress wave propagation in the vessel wall.

5. DISCUSSION

Thus, the model of pulls flow through an artery with a deformable wall was developed. Two principal positions were used as a basis for computer simulation: the mechanical model of blood and vessel. The blood was considered as a non-Newtonian liquid with Power law for correlation between the stress tensor of the strain velocity tensor. This law can describe the behaviour not only of liquids but also of suspensions and the blood too. But this law contains two parameters *n* and *k*, which are usually considered as constant. In fact, they can vary along the stream and cross-section, but the nature of this change has not been adequately studied. There are reasons to assume that these constants depend also on hematocrit [14], but there are only assumptions of specific kind of this dependence.

It is necessary to point out the impossibility to develop the vessel mechanical model containing the inner structure similar to the vessel wall structure. In this connection the authors are forced to use the averaged properties and consider the wall material as hyperelastic orthotropic and incompressible (for example), also this variant is not unique.

In general, it is necessary to point out that the main obstacle in the way of computer simulation of a medico-biological object and processes is not the absence of the mathematical instruments and computer technologies, but the lack of physiological knowledge and specific data, which are required for construction of a more adequate model.

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