Application of Fourier Transform in Optical Flow Method

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ABSTRACT

The problem of object motion detection by its images appears during processing of images in different fields. The article considers the family of methods for determining the velocity field in the assumption of the consistency of the objects' color and its derivatives. Mathematical formalization in the form of a system of second order differential equations is considered. Application of numerical methods, for the solution of these systems is presented. Implementation of method of Fourier transform is described. Application problems are discussed. Aperture problem and ways to solve it by combining different methods are discussed.

Keywords

Computer graphics, optical flow, Fourier transform.

1. INTRODUCTION

The article considers methods, which can be applied to optical flow detection. Optical flow is one of the object motion representations, where the velocity vector is associated with each point of the image, which corresponds to the point of the object. The article describes the mathematical formalization of one of the methods, discusses some difficulties of using these methods. Different ways of implementation of Fourier transformation are considered.

2. METHOD OF OPTICAL FLOW

Generally, the initial assumptions for method selection, which will be applied to some class of images, depend on different properties of these images. As in case of recognition problems, the external geometric similarity of one and the same object in different images can be applied.

There is method for motion detection in images using the Fourier transform for both images in analyzing pair which is based on search of shift by a phase correlation. Using this method in itself allows to detect the object shift in an image but it does not allow to detect the object rotation.

Smaller difference between moments, which are displayed in these images, is better for the selection of the object. In the easiest way, the independence of the object color on the time can be taken as a basis. But brightness of images can be different because of the quality of color rendition or illumination changes of the scene, so constancy of color gradient is used to improve the results. In a more common way, brightness of the image can be uneven, so constancy of the second partial derivatives, Hessian matrix or Laplacian are considered in some cases.

Assumed assumptions are insufficient for construction of equation system. Its description of connection between points of the images without some order and relationship between these points. In these assumptions there are ambiguity of the correspondence between points in different images. One of ways to solve this difficulty is assumption of smoothness of velocity flow. Its description of continuity of moving objects and time independence of objects structure. But in this assumption objects can change form and proportions.

3. PROBLEM FORMULATION AND EASIEST ALGORITHM OF OPTICAL FLOW BUILDING

At this part detailed description of family of optical flow methods will be described. In the first, method, which is based on assumptions of color constancy and smoothness of flow, will be formalized. In the second, algorithms for assumption of Hessian matrix constancy will be described [1].

We will describe images, where point color is represented as one number of brightness $I(\mathbf{x})$, where vector $\mathbf{x} = (x_1, x_2, t)$ includes coordinates and the time. Assumption of constancy of brightness is formulated in the form of minimization task

$$I_z = I(\mathbf{x} + \mathbf{u}) - I(\mathbf{x}) \to \min,$$

where vector $\mathbf{u} = \mathbf{x}'_t = (u_1, u_2, 1)$ is corresponded to velocity field, which is parameter of minimization. For the truth of this condition for each point of the image we will consider minimization task for integral

$$E_1 = \int_\Omega [I(\mathbf{x} + \mathbf{y}) - I(\mathbf{x})]^2 d\mathbf{x}.$$

We can add assumption of smoothness of velocity flow

$$E = \int_\Omega [I(\mathbf{x} + \mathbf{y}) - I(\mathbf{x})]^2 d\mathbf{x} + \alpha \int_\Omega \|\bigtriangledown u_1\|^2 + \|\bigtriangledown u_2\|^2 d\mathbf{x},$$

where α is regularization coefficient for our two assumptions. We will use sing $\bigtriangledown$ for gradient and sing $\triangle$ for Laplacian. For solution of minimization task we can construct system of Euler–Lagrange equation system:

$$I_z \bigtriangledown I(\mathbf{x} + \mathbf{u}) - \alpha \begin{pmatrix} \triangle u_1 \\ \triangle u_2 \end{pmatrix} = 0.$$

Let us give this expression to the system of linear equations. We construct an iterative scheme and introduce vector $d\mathbf{u}^k = \left(u_1^k, u_2^k, 1\right)^T$.

$$I_z^{k+1} \triangle I\left(\mathbf{x} + \mathbf{u}^k\right) - \alpha \begin{pmatrix} \triangle u_1^{k+1} \\ \triangle u_2^{k+1} \end{pmatrix} = 0.$$

For solution of the system we can use inverse iteration scheme. We can transform Laplacian by interpolation of flow value, which is received in previous iteration [2], and linear part of I_z^{k+1} can be received by Taylor formula, then our system takes next form

$$I_z^{k+1} \approx I_z^k + I_{x_1}^k du_1^k + I_{x_2}^k du_2^k.$$

Wherein $u_1^{k+1} = u_1^k + du_1^k,\ u_2^{k+1} = u_2^k + du_2^k$. As a result

$$\left(I_z^k + I_{x_1}^k du_1^k + I_{x_2}^k du_2^k\right) \bigtriangledown I\left(\mathbf{x} + \mathbf{u}^k\right) -$$

$$-\alpha \begin{pmatrix} \triangle u_1^{k+1} \\ \triangle u_2^{k+1} \end{pmatrix} = 0$$

So linear system for $\mathbf{u}^k$ was built. This system can be solved by different methods [3], for example, by fixed-point iteration method, by next scheme

$$\left(I_z^k + d\mathbf{u}^{k,l+1} \bigtriangledown I^k\right) \bigtriangledown I\left(\mathbf{x} + \mathbf{u}^{k,l}\right) -$$

$$-\alpha \begin{pmatrix} \triangle\left(u_1^k + du_1^{k,l+1}\right) \\ \triangle\left(u_2^k + du_2^{k,l+1}\right) \end{pmatrix} = 0.$$

When assumptions of constancy of Laplacian and flow smoothness are used, the result has next form

$$\sum_{i,j=1}^{2} \bigtriangledown I_{x_i x_j}^k \left[I_{x_i x_j z}^{k,l+1} + \left(\bigtriangledown I_{x_i x_j}^k\right)^T d\mathbf{u}^{k,l+1}\right] -$$

$$-\alpha \begin{pmatrix} \triangle\left(u_1^k + du_1^{k,l+1}\right) \\ \triangle\left(u_2^k + du_2^{k,l+1}\right) \end{pmatrix} = 0.$$

Here partial derivatives are denoted as subscripts of $x_i x_j$.

4. USING OF FOURIER TRANSFORM FOR SOLUTION OF EQUATION'S SYSTEM

There are two approaches for improvement of accuracy and velocity of convergence of optical flow method by Fourier transform using least.

First of them is applied for searching of solution of linear equation system [3], two examples of this systems was built in previous segment. Such system contains Toeplitz matrix. For solution of equation system of this type can be accelerated by fast Fourier transform.

Second of them is applied in common way for optical flow method with assumption of flow smoothness. In this method task of detection of vector field distribution and concentration of brightness particles n is posed.At this case the vector filed is potentially, it can be represented as gradient of some filed ϕ:

System without boundary conditions can be formulated in next form [4]:

$$\frac{\partial n}{\partial z} + I_{x_1} \frac{\partial n}{\partial x_1} + I_{x_2} \frac{\partial n}{\partial x_2} = 0,$$

$$\frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2} = 0,$$

5. EXAMPLE OF OPTICAL FLOW

Realization of optical flow method can be produced in different way, but all of them have common features [5]. As a rule, input image is processed by smoothness filter. Pyramid of images can be applied for increase of convergence rate. In basis of the pyramid original size image is located and each next level of the pyramid contain image with half the size.

As an example shift to vector (1,1) of nature image pic. 1 will be considered.

Figure 1. Example image

Optical flow for the image is showed at pic. 2.

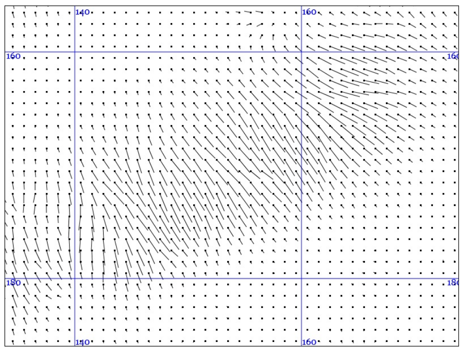

Figure 2. Optical flow for example image

Some problems of optical flow method are showed here. Velocity for each point of the image can not be detected, special, for regions off small color changes.

6. CONCLUSION

At the article methods, which can be applied for optical flow detection, are described. The methods can be use in different fields, but partial features should be taken into account. In common more detail information about analyzed images allows to construct more accurate algorithm for optical flow detection.

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