Energy Efficient WSN Target Coverage Heuristics

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ABSTRACT

The problem of energy aware wireless accessibility from a collection of statically positioned sensor nodes to a given set of target nodes is considered. The simple plain-disc-model with the sensing radius ρ_s of wireless nodes is well known. Constant guarantee approximation algorithm is known for the ideal disk model but we will continue the study of general model in the case with hindrances. This model is defined by the use of matrices of sensor-target accessibility. The traditional connectivity issue of WSN is out of the scope of current research and the focus is on *coverage*. We aim at applying the widely-decentralized time-sharing model, where sensors collectively share the duty of continuous covering of the total collection of the set of targets. That is, when part of the nodes may accept for a time interval the sleep regime, minimizing in this way the energy consumption. In an easy step we obtain, that the mathematical problems arisen are related to the well-known combinatorial set cover problems. Set cover is one of the typical NP complete problems, which means that our solution will likely be not exact, but – approximate, or even heuristic. We bring analysis of the theoretical resources around these postulations. An extension of Integer Linear Programming model is implemented and demonstrated, being applied on the WSN domain coverage issues. In a complementary manner, and for the first time, we manage the appearing covering structures using the terms of monotone Boolean functions (the main result).

Keywords

WSN, ILP, MBF

1. INTEGER PROGRAMMING MODEL OF THE WSN TARGET COVERAGE PROBLEM

Several sustained fundamental models, concentrated around the issues of – target coverage, sensor connectivity and energy optimization are developed in wireless sensor network (WSN) research area [1]. These models, as a rule, use simplifications such as supposition that the WSN duty occurs on an ideal plain, or they suppose the simple unit disk model – when the radio accessibility from the sensing nodes is bounded by identical circles of radius 1.

Current state of research on WSN Target Coverage includes a large set of publications. Our initial starting point is the paper [2] that brings the basic interpretation in terms of integer and linear programming. Of special interest are the results about the constant guarantee covering in the plain case [20], but we consider the case of general accessibility given by arbitrary matrices, and this model is broader than the ideal plain model.

In formal level [2], we are given:

- a set of n sensor nodes $\mathcal{S} = \{\mathrm{s}_1, \dots, \mathrm{s}_i, \dots, \mathrm{s}_n\}$,
- a set of m target points $\mathcal{R} = \{\mathrm{r}_1, \dots, \mathrm{r}_k, \dots, \mathrm{r}_m\}$,
- sensing radius of sensors is equal to $\rho_s = 1$, and the current sensor lifetime is given by the constant parameter l_i, which can be normalized to the value 1.

The covering relationship between sensors and target points depend on their geographical allocation (2-3D, landscape, radio/sensing areas). In a simple model each sensor covers all target points that belong to the unit disc around the sensor location. And each target point is covered by each sensor that belongs to the unit disc around that target, see Figure 1. We accept condition that each sensor covers at least one of the targets, and that each target can be covered by one of the sensors (C1). Otherwise, we mark these points as outlier elements and delete them from the model.

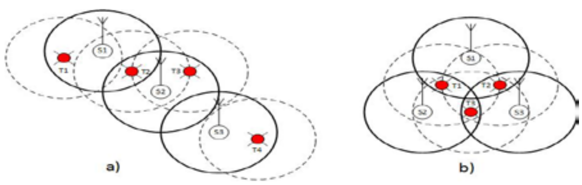

Figure 1. Two example of WSN deployments.

Which are the typical decentralized algorithmic actions of sensors and targets after their deployment? It seems that it is enough that each target selects one sensor of its disc (sensing radius disc of sensors) as a covering sensor. The total set of selected sensors is a covering set but it can be redundant. Is it possible to delete the unnecessary sensors so that each target becomes covered by a unique sensor (C2). Figure 1(a) shows that for some schemes this is not the case.

Analytical definition of the sensor–target coverage model can be given by the set of coordinates of these elements, as well as by analytical definition of radio wave parameters and sensing hindrances, if any. However, more simply, the task of reachability of target from sensors may be given with the help of (0,1) matrices and the corresponding bipartite graphs of these schemes.

Consider the general sensor-target covering relationship given by the coverage matrix $\mathcal{M}$ in Figure 2. Rows $i \in \overline{1,n}$ of the matrix $\mathcal{M}$ correspond to the sensing elements $\mathrm{s}_i \in \mathcal{S}$, also representing subsets of the finite set of targets, $\mathcal{R}$, covered by sensors s_i.

$$\begin{array}{c|c|c|c|c|c|} & r_1 & \dots & r_k & \dots & r_m \\ \hline s_1 & a_{1,1} & \cdots & a_{1,k} & \cdots & a_{1,m} \\ \hline \vdots & \dots & \dots & \dots & \dots & \dots \\ \hline s_i & a_{i,1} & \cdots & a_{i,k} & \cdots & a_{1,m} \\ \hline \vdots & \dots & \dots & \dots & \dots & \dots \\ \hline s_n & a_{n,1} & \cdots & a_{n,k} & \cdots & a_{n,m} \\ \hline \end{array} \qquad (1)$$

Figure 2. Matrix $\mathcal{M}$ of sensor-target covering relationship.

In this sensor-target coverage matrix $\mathcal{M}$, $a_{i,k} \in \{0,1\}$, and $a_{i,k} = 1$ if and only if the sensor s_i covers the target point r_k. As we already mentioned, $\mathcal{M}$ can't be an arbitrary matrix

– its structure depends on the modelling properties – geometry, plain model, disk model, etc.

We consider the Target-Coverage Energy-Aware model with the following properties and requirements:

- The time-shared control mode is being applied to sensor activities, which means that sensors may be operational during a number of time intervals, - with the total duration that do not exceed the time frame l_i. It is convenient to normalize initial values of l_i up to the 1.
- At each time point when the set of sensors that are active – and in integration they cover the set of all target points – we say that the set of sensors is *covering*. Covering sub-sets, acting in different time frames may intersect by some of the sensors used. If they do not intersect, then they may last their activity full l time duration each. The case of intersecting sets makes it more flexible the use of total work time l but mathematically this is related to complex tasks, complex algorithms and optimisation issues.

Compose subset of sensors that cover the individual target points, t_k: $\mathcal{S}_k = \{i|$ sensor s_i covers target $\mathrm{t}_k\}$ which, as we see, is the set, coded by the k^{th} column of matrix $\mathcal{M}$ in Figure 2. We suppose that the subsets $\mathcal{S}_k$ are not empty. $\mathcal{S}_k$ is for one-target, s_k. Alternatively, we are interested in knowing collections of subsets $\mathcal{S}$, that cover collectively the set of all-targets. Having such subset of $\mathcal{S}$ and activating its sensors we provide the target coverage service in a time frame, that may last as long as the lifetime of individual sensors, in this collection. Applying one or several stages of services of this kind still it can remind additional energy of some sensors, that is not used by the already selected and applied subsets of $\mathcal{S}$. The problem is in composing not one but as many as possible target covering collections, and running them in a sequential time sharing manner. In a more realistic mode, we have to restrict the activation time of individual collections. This splits the lifetime of an individual sensor among the covering collections, making it more flexible the coverage in a time-shared manner. The traditional mathematical formulation of the considered problems given in [2] is based on framework of Minimal Set Cover (MSC), interpreted in terms of the Integer Programing (IP) problem. The resulting model becomes not only integer valued, which brings known difficulties in stage of optimization, but it is also nonlinear, entering in this way the domain of general Lagrangean relaxation and the related heuristics.

We continue this research line. The first group of models considered in this paper is related to the same well-known combinatorial MSC problem. The second part of our models uses the classic ILP technique, and then, the Energy Minimized Target Coverage problem is considered in the form of a composite MSC+ILP procedure. Used separately, these procedures have pros and contras against the algorithmic complexity and accuracy issues. In combination they give more flexible toolsets to accommodate the problem restrictions to the application needs. Additively, we will incorporate this technique together with one more integrative modelling element – the formalism of Monotone Boolean Function (MBF), which will bring several sensitive and additional benefits. Let us bring the necessary terms and definitions.

1.1 MSC problem

A set system, or hypergraph, consists of a finite base-set Σ and a collection F of subsets of Σ (sets of vertices and edges of the hypergraph). Set cover is a collection $f \subseteq F$, which in integration $\bigcup_{\sigma \in f} \sigma$ covers the basic set Σ. Set cover optimization problem requires building a minimum-capacity ($\min|f|$) coverage. It also considers the structure of the set of all set covers, its description, generation, and complexity issues.

Set cover appears as a data model in extremely large number of applications so that even being NP complete it requires to find ways of composing solutions to it – be it approximate, partial or heuristic. Back in 1974 Johnson [6] proved that the greedy (gradient) algorithm for this problem provides $\ln n$ approximation, where n is the size of the base set (this result is then extended by Chvatal [7] to the weighted version of the set cover problem). In 1975 Lovasz [8] has built a linear programming relaxation, which also provides the $\ln n$ approximation factor to MSC.

Different types of constrained set cover problems appear in research domain. Call the frequency of element $\sigma \in \Sigma$ the number of subsets containing that an element. It is known that the gradient algorithm gives an approximation also to this type of degree-weighted set cover problems [9-11].

The WSN target coverage set cover interpretation is specifically related to the plane geometry in the sense that the appearing structures include sensor positioning, sensor sensing discs, the combined sensor-target Voronoi diagrams [12], and other geometric relations.

1.2 Monotone Boolean functions

"The general knowledge discovery process is not always easy or efficient, and even if knowledge is produced it may be hard to understand, interpret, validate, remember, and use. Monotonicity is a pervasive property in nature: it applies when each predictor variable has a non- negative effect on the phenomenon under the study. Due to the monotonicity property, being able to observe the phenomenon under specifically selected conditions may increase the accuracy and completeness of the knowledge at a faster rate than a passive observer …" [13]. Monotone Boolean functions are the modelling technique of set cover problems in general, and in WSN devoted issues, specifically.

Let E^{n} denote the set of all binary vectors of length n. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta)$ be such vectors. Then, the vector $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ precedes $\beta = (\beta_1, \beta_2, \ldots, \beta)$ (denoted as: $\alpha \preccurlyeq \beta$) if and only if the following is true: $\alpha_i \preccurlyeq \beta_i$, for all $i \in \overline{1,n}$. If, at the same time: $\alpha \neq \beta$, then it is said that α strictly precedes β (denoted as: $\alpha \prec \beta$).

A Boolean function f(x) is monotone if for any vectors $\alpha, \beta \in \mathrm{E}^{\mathrm{n}}$, the relation $f(\alpha) \preccurlyeq f(\beta)$ follows from the fact that $\alpha \preccurlyeq \beta$. Let M_n be the set of all monotone Boolean functions defined on n variables. A binary vector of length n is said to be the upper zero of function $f \in \mathrm{M}_n$, if $f(\alpha) = 0$ and then, for any vector β such that $\alpha \prec \beta$ we have $f(\alpha) = 1$. In a similar way the lower unit of a monotone function f can be determined.

Two issues are of our interest concerning the use of MBF.

a) Consider an arbitrary collection Σ of subsets of the sensor set $\mathcal{S} = \{s_1, \ldots, s_i, \ldots, s_n\}$ in the following way. $\Sigma = \{\tilde{\sigma}_1, \ldots, \tilde{\sigma}_j, \ldots, \tilde{\sigma}_p\}$, $p \leq 2^n$ and it represents a subset of the binary cube E^n (power n-set). For a $\tilde{\sigma}_j = (\sigma_{j,1}, \sigma_{j,2}, \ldots, \sigma_{j,n}) \in E^n$ compose the set of subsets of $\mathcal{S}$ indicated by all coordinate values 1 in $\tilde{\sigma}_j$. Some of these sets cover the entire set of targets, $\mathcal{R}$. If α causes a cover, then β, that obey $\alpha \preccurlyeq \beta$, is also a cover. Denote the MBF received in this way by f_Σ. Let α be a lower 1 of the f_Σ. Its role is specific in cover because of it is the minimal necessary collection of covering sensor collections that covers the set $\mathcal{R}$. The WSN coverage problem, as we see, is composed of two layers – we choose collections of sensors in a way that they, in integrity, cover the target domain. The covering subsets are the same $\tilde{\sigma}_j$ but represented not only by the sensors that belong to the set $\mathcal{S}$, but alternatively by the subsets of target points of $\mathcal{R}$, that are covered by the elements of $\tilde{\sigma}_j$. MBF, corresponding to this structure, we may denote by the same symbol f_Σ.

b) Given the matrix (1), procedure of constructing of function f_Σ asks for several steps. At first, it is to determine the proper set Σ of initial sensor cover subsets. One may want to choose all subsets, coded by lower units of f_Σ. There are 2 ways of achieving this goal: constructing the whole f_Σ which is time and memory consuming for large networks, and, alternatively, a direct or approximate construction and reinforcement of the proper collections of subsets. These algorithms are known as query-based MBF reconstructing algorithms, they are hard and they are studied intensively and broadly [13]. Korobkov V. [15] described the basic relations among the set of all monotone Boolean functions. Korshunov A. [16] derived the asymptotic formula for the number of n-dimensional MBF. Hansel G. [17] invented the famous chain split of E^n proving the exact lower bound complexity of the MBF reconstructing. Tonoyan G. [18] was the first describing an effective algebra for computations over the Hansel's chains. Afterward Sokolov D. [19] simplified the chain split so that the computation algebra itself became more transparent. Other related works can be found at [14].

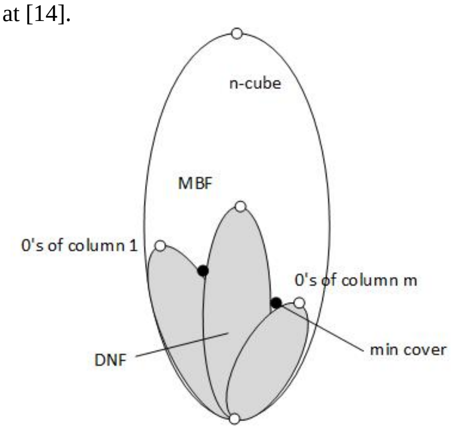

Figure 3. Power set with all set covers in accord with the matrix of Figure 2. The bottom point represents the empty set of sensors. "0's of column i" means the sensor subset that includes all sensors, marked by 0 in column 1 of matrix in Figure 2. This point, and all vertices below it can't be a covering set. DNF is disjunction of all these subcubes. Any point above the DNF is a covering set. Negation of DNF corresponds to monotone Boolean function (MBF) describing the set of all covering sets and "min cover" is one of the lower 1's of this function.

The MBF reconstructing technique may help with the direct search of the lower units of function f_Σ. The working schema supposes, that there exists an oracle Ω_Σ that knows the function f_Σ. With the help of a certain number of accesses to the oracle, and using the Hansel's chain split technique and its extensions, several procedures might be devised and applied in determination of several or all lower units of MBF.

Fundamentals of the MBF reconstructing algorithms developed in [15, 17-20] introduce us the following lessons. There exists an incredible chain-split of E^n with the very basic property of relative completion. Chains are symmetrically positioned against the layers of the cube E^n. Reconstructing algorithms of [15, 17-20] essentially use the chain-split construction.

1.3 General terms

Consuming the given short description above, let us formulate the postulations directly in terms of the WSN energy minimization target-coverage model. Consider the time-sharing table with the target covers $\tilde{\sigma}_1, \dots, \tilde{\sigma}_i, \dots, \tilde{\sigma}_p$:

$\tilde{\sigma}_1$	...	$\tilde{\sigma}_j$	...	$\tilde{\sigma}_p$		
$x_{1,1}$	$\cdots$	$x_{1,j}$	$\cdots$	$x_{1,p}$	s_1	
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\vdots$	
$x_{i,1}$	$\cdots$	$x_{i,j}$	$\cdots$	$x_{i,p}$	s_i	(2)
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\vdots$	
$x_{n,1}$	$\cdots$	$x_{n,j}$	$\cdots$	$x_{n,p}$	s_n	
l_1		l_2		l_p		

Figure 4. The time sharing table of target covers.

Here $l_1, \dots, l_j, \dots, l_p$ define the time duration of active regime of subsets $\tilde{\sigma}_j$. $x_{i,j}$ are indicators, binary variables, of involvement of sensors s_i into the cover-subsets $\tilde{\sigma}_j$.

Theorem In target-coverage Figure 4., any replacement of sensor-covers $\tilde{\sigma}_j$ by lower units of f_Σ (these are parts of covers $\tilde{\sigma}_j$) will draw to solution enhancement, minimizing energy consumption and maximizing the life time of the system.

This postulation is not about an obligatory enhancement, because of the sparing energy depending on (1) and (2), in principle, may be unable to extend an additional time frame for covering all targets. Henceforth, the note is about the exceptional and unique cases. But in general, this change saves energy that potentially and in a proper management, may frequently help to approach the optimal functionality.

1.4 ILP problem

One convenient interpretation of combinatorial set cover problem is through the Integer Linear Programming model. Large combinatorial optimization problems are often formulated in terms of mathematical programming, which is not necessarily linear. Further relaxation in some cases reduces these problems to the Linear or Integer Linear programming models. Other possible heuristics include random search procedures, sub-gradient optimization and other computational techniques. ILP is one of the basic NP-complete problems. A large number of combinatorial problems have their polynomial reductions to ILP. Having this knowledge on MSC and ILP, we want to develop appropriate approximation models and algorithms for WSN domain. In fact, first we model the MSC problem as an Integer Programming and then we use the relaxation technique to design a Linear Programming-based model to solve the MSC.

Integer Programming Formulation of the MSC Problem

Following [2] let us set a threshold p for the total number of the set-covers used in the model. We formulate the MSC problem as follows: introduce matrix X, whch presents the p number of used subsets of sensors. $l_1, \dots, l_j, \dots, l_p$ define the time duration when the subset $\mathcal{S}_j$ of sensors is active. We present all this information together in the table form (1)+(2).

Variables: $a_{i,k}$ are binary constants characterizing sensor-target coverage relations, $x_{i,j}$ are Boolean variables, for $i = \overline{1,n}$ and $j = \overline{1,p}$; $x_{i,j} = 1$ if sensor s_i is a member of the target covering set $\tilde{\sigma}_j$, otherwise $x_{i,j} = 0$, l_j are nonnegative reals, $0 \le l_j \le 1$ for $j = \overline{1,p}$ representing the time allocated for the target cover $\tilde{\sigma}_j$.

The optimization problem about the sensor cover of targets can be written as:

$$Maximize\ l_1 + \cdots + l + \cdots + l_p$$

$$\text{subject to } \sum_{j=1}^{p} x_{i,j} l_j \le 1 \quad \text{for all } s_i \in \mathcal{S}$$

$$\sum_{i=1}^{n} a_{i,k} x_{i,j} \geq 1 \text{ for all } k = \overline{1,m}, \text{and } j = \overline{1,p} \quad (4)$$

Remarks:

- the first constraint, $\sum_{j=1}^{p} x_{i,j} l_j \leq 1$ for all $s_i \in \mathcal{S}$ guarantees that the time allocated for each sensor s_i, across all p target covers, is not larger than 1, which is the life time of each sensor.
- the second constraint, $\sum_{i \in \mathcal{S}_k} x_{i,j} \geq 1$ for all $r_k \in \mathcal{R}$, $\mathcal{S}_k = \{i|$ sensor s_i covers target $r_k\}$, and $j = \overline{1,p}$ guarantees that each target r_k is covered by at least one sensor s_i in each target cover $\tilde{\sigma}_j$.

We observe that the term $x_{i,j} l_j$ *is* not linear so that (4) is not a linear program (LP). Even if all l_j are equal, and constant, $x_{i,j}$ are Boolean so the reduced (4) remains an ILP.

2. WSN TARGET COVERAGE TECHNIQUE AND SOLUTIONS

2.1 LP-MSC-1 Heuristic

Transform the model (4) into an LP [2] and further apply the relaxation technique:

$$Maximize\ l_1 + \cdots + l_j + \cdots + l_p$$

$$\text{subject to } \sum_{j=1}^{p} y_{i,j} \leq 1 \quad \text{for all } s_i \in \mathcal{S}$$

$$\sum_{i=1}^{n} a_{i,k} y_{i,j} \geq l_j \text{ for all } r_k \in \mathcal{R}, \text{and } j = \overline{1,p} \quad (5)$$

where $0 \leq y_{i,j} \leq l_j \leq 1$.

<u>LP-MSC-1 Heuristic</u> (integer nonlinear programming + Lagrangean relaxation + combinatorial approximation and iterations):

<u>Step 1. Initialization</u> Solve the linear programming LP (5) formulated above. Let $(y^*_{i,j}, l^*_j), i = \overline{1,n}, j = \overline{1,p}$ be the optimal solution of LP. Set the network lifetime $G = 0$.

Note that all $\tilde{\sigma}_j$ obtained in this way are valid target covers. If to suppose that l_j are even some very small nonzero values then evidently the values $y_{i,j}$ satisfying (5), to its second constraint, generate a target cover $\tilde{\sigma}_j$. $\tilde{\sigma}_j$ can be further optimized by deletion of unnecessary sensor elements from it. After that, intuitively, the problem is to construct a table $y_{i,j}$ with more or less equal number of nonzero entities. The ideal matrix will be the one with columns corresponding the minimal "ones" of the covering monotone Boolean function, and regular in rows – that is, having the same row weight. There are two ways to obtain this. One is the hard way of constructing all minimal "ones" of MBF with further selection of a subset of minimal ones with almost regular rows. The second way may construct a large set of target covers by LP (5), considering then its intersection with a properly selected layer of n-cube (or a direct random selection on the n-cube layer). Intersection provides an equal number of sensors at this stage. The overhead is due to presence of additional sensors that may be eliminated, and this will move the element to a lower layer increasing in this way the potential life time of the system. Anyway, it is an important remark, that the LP (5), in polynomial time, solved for a large parameter p, can give as a large set of covering sensor collections as the basis of further optimization of their time intervals.

<u>Step 2. Fitting</u> The initial approximate solution can be obtained in the form of the set $(y^*_{i,j}, l^*_j), i = \overline{1,n}, j = \overline{1,p}$ as follows:

for all $j = \overline{1,p}$ **do**

/* this cycle considers each target cover $\tilde{\sigma}_j$ */

set $\boldsymbol{y^0_{i,j} = 0}$

/* in the cover $\tilde{\sigma}_j$ for all sensors $s_i \in \mathcal{S}$ */

set $\boldsymbol{l^0_j = 0}$

for all $k = \overline{1,m}$ **do**

$i^{\wedge}_{jk} \in arg \max_{i \in \mathcal{S}_k} y^*_{i,j}$

/* for current target cover $\tilde{\sigma}_j$ and all target $r_k \in \mathcal{R}$ */

set $y^0_{i^{\wedge}_{jk},j} = y^*_{i^{\vee}_{jk},j}$

end for

$i^{\vee}_j \in arg \min_k i^{\wedge}_{jk}$

set $l^0_j = y^0_{i^{\vee}_{jk},j}$

end for

This step selects the best possible durations l^0_j of covers $\tilde{\sigma}_j$ by the current solution of (5), $l^0_j = \min_k \max_{i \in \mathcal{S}_k} y^*_{i,j}$ and $y^0_{i,j}$ be set equal to 0 or l^0_j such that for every r_k, there exists an $i \in \mathcal{S}_k$ such that $y^0_{i,j} = l^0_j$. Denote $\rho = \max_{r_k \in R} |\mathcal{S}_k|$. Then, $l^*_j \leq \sum_{i \in \mathcal{S}_k} y^*_{i,j} \leq \rho \max_{i \in \mathcal{S}_k} y^*_{i,j}$, for all $r_k \in R$, therefor, $l^*_j \leq \rho \min_{r_k \in R} \max_{i \in \mathcal{S}_k} y^*_{i,j} = \rho l^0_j$. Due to relaxation used in (5), the real optimal solution $l^{\underline{0}}_j$ obeys relation $l^{\underline{0}}_j \leq l^*_j$ so that $l^0_j \geq 1/\rho \cdot l^{\underline{0}}_j$. Hence, $\sum_{j=1}^{p} l^0_j \geq 1/\rho \cdot \sum_{j=1}^{p} l^{\underline{0}}_j$.

After the first approximation:

- each sensor s_i, $i = \overline{1,n}$, has a remaining life time $l_i = 1 - \sum_j y^0_{i,j}$
- network lifetime $G = G + \sum_j y^0_{i,j}$.

<u>Step 3. Iterations</u> We iteratively repeat Step 1 and Step 2 by solving the updated linear program (5), in order to improve the current network lifetime G. The iteration is executed while each target is covered by at least one sensor having the remaining lifetime greater than 0.

Finally, at this step, the LP-MSC-1 heuristic returns the network lifetime approximation G.

2.2 LP-MSC-2 Heuristic

Starting from this point of the work we form a different than the LP-MSC-1 Heuristic [2], based on analysis of the combinatorial algorithms considered above. The basic idea is in how to increase and properly use the value $\min_k \max_{i \in \mathcal{S}_k} y_{i,j}$.

In one case we insert a new parameter ε, and consider an LP of maximizing this parameter:

$$Maximize\ \varepsilon$$

$$\text{subject to } \sum_{j=1}^{p} y_{i,j} \leq 1 \quad \text{for all } s_i \in \mathcal{S}$$

$$\sum_{i=1}^{n} a_{i,k} y_{i,j} \geq \varepsilon \text{ for all } r_k \in \mathcal{R}, \text{and } j = \overline{1,p} \quad (6)$$

where $0 \leq y_{i,j} \leq 1$.

<u>LP-MSC-2 Heuristic</u> (LP + Step 2. of LP-MSC-1 + iterative LP + combinatorial approximation):

The new LP-MSC-2 Heuristic solves the LP (8) first, and then it applies Step 2. described above. The optimal solution obtained in this way evidently equals to $p\varepsilon$. And, if after the optimization still some time duration remains in sensors, a step similar to Step 3. may be applied to refine the final solution. The idea of LP-MSC-2 is to directly address the narrower point of the model – the requirement to provide a maximal value to the forms $\min_k \max_{i \in \mathcal{S}_k} y_{i,j}$. Here $y_{i,j}$ is directly the activation time duration of the sensor i in coverage j. We lose when several sensors covering the same target have to be active in a time being. In this case only a part of the ε will be transferred into the coverage duration.

Simulation results will compare the considered heuristics LP-MSC-1 and LP-MSC-2.

The Result of the paper. Finally, let us describe the basic heuristic approaches, developed in this paper, LP-MBF-3. LP-MBF-3 is based on the better use of the Lagrangean relaxation model given above, and on use of error correcting codes and the monotone Boolean functions. In the first stage, let us a consider a procedure of a general purpose. Let in some heuristics an approximate solution of the problem be found in terms of the pairs $(x_{i,j}, l_j)$. Columns x^j present covering subsets of sensors. Durations l_j are found iteratively, which can be not the best choice, and the covering subsets might be not optimal (deadlock).

2.3 LP-MSC-3 Procedure

Here the following strategy might be of interest. At the first stage, let us minimize the covering subsets by consecutive steps of deletion of unnecessary sensors, if there are any. This can be done under different strategies, but we will not consider the issue in detail here due to room restrictions. At the second stage, when we have an optimized covering collection in the form of a matrix consisting of values $x_{i,j}$ we apply again the linear programming. Here, as the second constraint of (4) is satisfied, it remains to solve the simplest LP with given (constant $x_{i,j}$), targeting to maximize the sum $l_1 + \cdots + l_j + \cdots + l_p$, and having to obey only the first constraint of (4). The exact solution of this formulation might exceed the step by step optimization result. Besides, this procedure is applicable to covering matrices $x_{i,j}$ that are constructed in any way, be it combinatorial, random, LP or other.

2.4 LP-MBF-3 Heuristic

Let us describe schematically the general structure of this approach, which is complicated in its nature. The idea is to construct sets of covering subsets and then apply a regular LP to determine the weights of covers - time intervals of covering set activities. Unlike the previous models, here the covering sets are searched for by combinatorial algorithms. Evenly sparsely distributed collections of Hansel chains are constructed and on these chains the lower covers are determined. Here the series of chain dichotomies and checks for covering are used (see the point 1.2). Further, as already mentioned in LP-MSC-2, weights of sets are determined by the use of ordinary linear programming algorithms.

An additional comment is required concerning the sizes of the covering subsets that appear in the model. This number is comparable to the number of target points and cannot be sharply higher than this. In terms of n-cube we seek for vertices that belong to the lower layers of E^n. This information is to be correlated to the chain collection procedure. Chains can be composed using constant-weight codes, randomly, or in some other ways.

3. CONCLUSION

WSN Target Coverage is a spatial-temporal type problem, mostly considered in 2-3D cases. For ideal problems, where there are no hindering elements to the radio penetration, a constant factor solution to this problem is known. The general form problem is given with the help of (0,1) accessibility matrices, so the problem remains complex. Additionally, to LP models and approximations, which are well known, this paper considers innovative mechanisms based on monotone Boolean functions and the corresponding approximations are presented in tight integration with those achieved through mathematical and especially the integer linear programing formalisms.

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