L_1 Adaptive Control of Quadcopters

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ABSTRACT
The issue of developing a control system for compensating external disturbances acting on the quadrocopter is considered. The proposed design procedure consists in introducing a decoupling compensator into the system and subsequent synthesis of an adaptive regulator that provides compensation of external disturbances. It is shown that by virtue of choosing the value of the adaptation gain one can reduce the control system error to any arbitrary small value. The proposed control architecture can be used for developing fault-tolerant control systems of multirotor copters.

Keywords
Quadcopter, multivariable control system, matrix decoupling compensator, L_1 adaptive regulator, state predictor, external disturbances, adaptation gain.

1. INTRODUCTION
Unmanned aerial vehicles (UAVs) are widely used in military and various civilian areas. In the latter case, they are used in: controlling traffic; assessment of the state of trunk pipelines and high-voltage transmission lines; monitoring the technical condition of buildings and other structures, as well as railways and roads; detection of fires in forests and peatlands; technical support in agricultural works and geological exploration, etc. [1-4].

In this article, the issue of development of L_1 adaptive control system [5, 6] of the angular motion of the quadcopter is considered, taking into account the external disturbances [7].

2. DYNAMIC EQUATIONS OF QUADCOPTER
A schematic representation of a quadcopter is shown in Fig. 1, where $O^I X^I Y^I Z^I$ - is the inertial frame (IF), in which the motion of the centre of mass of the quadcopter is described; $OXYZ$ - is the quadcopter's body fixed frame with the origin in the centre of mass [7], θ, ϕ и ψ - are the pitch, roll and yaw angles, and L is the distance of each motor from the centre of mass O [7].

In the general case, the translational motion of the centre of mass of the quadcopter with respect to the IF and angular motion of the quadcopter in the body fixed frame are described by the following nonlinear equations of the sixth order [2, 7]:

$$m\frac{d^2x}{dt^2} = u_z(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) + \gamma_x, \quad (1)$$

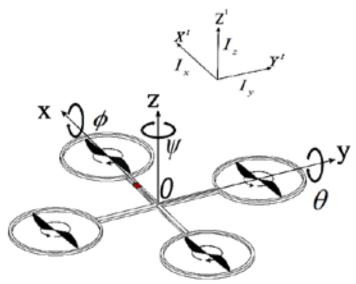
$$m\frac{d^2y}{dt^2} = u_z(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) + \gamma_y, \quad (2)$$

$$m\frac{d^2z}{dt^2} = u_z(\cos\phi\sin\theta\cos\psi) - mg + \gamma_z, \quad (3)$$

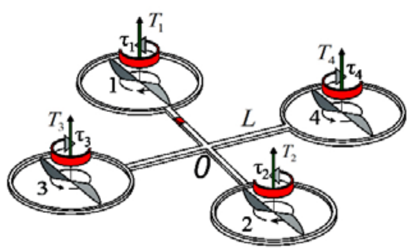
$$I_x\frac{d^2\theta}{dt^2} = u_\theta + \gamma_\theta + (I_y - I_z)\frac{d\phi}{dt}\frac{d\psi}{dt} - J_T\frac{d\phi}{dt}\Omega, \quad (4)$$

$$I_y\frac{d^2\phi}{dt^2} = u_\phi + \gamma_\phi + (I_z - I_x)\frac{d\theta}{dt}\frac{d\psi}{dt} - J_T\frac{d\theta}{dt}\Omega, \quad (5)$$

$$I_z\frac{d^2\psi}{dt^2} = u_\psi + \gamma_\psi + (I_y - I_z)\frac{d\theta}{dt}\frac{d\phi}{dt}. \quad (6)$$



a)



b)

Fig. 1. Schematic representation of the quadcopter:
a – coordinate systems, b - motors locations

In these equations, m is the mass of the quadcopter; g - the gravitational constant; I_x, I_y, I_z - the moments of inertia with respect to OX, OY, OZ axes; u_z - the lift force along the vertical axis $O^I Z^I$; u_θ, u_ϕ, u_ψ - the control thrusts around the principal axes of inertia; γ_x, γ_y, γ_z – and

γ_θ, γ_ϕ, γ_ψ - the unknown external disturbances (for example, wind gusts); J_T - the moment of inertia of each motor; Ω , the overall speed of propellers:

$$\Omega = -\Omega_1 - \Omega_2 + \Omega_3 + \Omega_4 , \tag{7}$$

where Ω_i is the i th propeller speed.

Note that all variables in the equations (1) - (7) are functions of time t. However, for brevity, the dependence of these variables on t is not explicitly indicated.

The main feature of the quadcopter control system is that there are only four control signals, namely the voltages u_1, u_2, u_3 and u_4 at the inputs of the motors, and the generated thrusts T_i of all four motors are directed parallel to the axis OZ. Such configuration does not give an opportunity to control all six degrees of freedom of the quadcopter. Therefore, as four controllable parameters of the motion of the quadcopter are usually chosen the angles θ, ϕ, ψ and the altitude z. The control of the movements in the horizontal plane $O^I X^I Y^I$ is carried out by changing the angles of roll θ and pitch ϕ [2, 7].

As shown in [7], in case of low angular velocities of the quadcopter and small pitch θ and roll ϕ angles, and assuming that during the linear movement the yaw angle ψ is zero, the controlled movements along OZ axis, as well as rotational movements, are approximately described in operator form, taking into account the dynamics of the motors, by the following linear equations:

$$z = \frac{1}{ms^2}\left[w_M(s)(u_1 + u_2 + u_3 + u_4) - mg\right] + \frac{1}{ms^2}\gamma_z , \tag{8}$$

$$\theta = \frac{L}{I_x s^2} w_M(s)(u_1 - u_2) + \frac{1}{I_x s^2}\gamma_\theta , \tag{9}$$

$$\phi = \frac{L}{I_y s^2} w_M(s)(u_3 - u_4) + \frac{1}{I_y s^2}\gamma_\phi , \tag{10}$$

$$\psi = \frac{K_\psi}{I_z s^2} w_M(s)(u_1 + u_2 - u_3 - u_4) + \frac{1}{I_z s^2}\gamma_\psi , \tag{11}$$

where $w_M(s)$ describes the dynamics of motors in the form of a first order aperiodic transfer function:

$$w_M(s) = \frac{T_i}{u_i} = K_M \frac{\omega_M}{s + \omega_M} \qquad i = 1,2,3,4. \tag{12}$$

Note that the constant coefficients K_ψ in (11) and K_M, ω_M in (12) are constructive parameters of motors, where the value ω_M is inversely proportional to the electromechanical time constant of the motor T_M, i.e., $\omega_M = 1/T_M$ [7]. Note also that all variables in (8)-(12) depend on the Laplace operator s, which is not explicitly indicated for brevity.

Introducing the vectors of controllable parameters $\bar{\eta} = [z, \theta, \phi, \psi]^T$, voltages at the inputs of the motors $\bar{u} = [u_1, u_2, u_3, u_4]^T$, external disturbances $\bar{\gamma} = [\gamma_z, \gamma_\theta, \gamma_\phi, \gamma_\psi]^T$, and the vector $F = [1\ 0\ 0\ 0]^T$, the equations (8) - (11) can be written in the following matrix form:

$$\bar{\eta} = W_U(s) R \bar{u} + W_\gamma(s)\bar{\gamma} - \frac{1}{s^2} F g , \tag{13}$$

where the matrices $W_U(s)$ and $W_\gamma(s)$ are diagonal [5], and the numerical matric R is equal to

$$R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix} \tag{14}$$

and characterizes the above kinematic features of the quadcopter.

Thus, the quadcopter control system belongs to multivariable control systems [8, 9], where rigid cross-connections between separate channels are characterized by a square numerical matrix R (14).

3. L_1 ADAPTIVE SYSTEM

Consider L_1 adaptive control system [5] of compensation of external disturbances $\bar{\gamma}$ caused by wind gusts. In accordance with the design procedure proposed in [7], let us introduce into the quadcopter control system a static decoupling compensator of the form:

$$\mathrm{K}_c = R^{-1} = \begin{bmatrix} 0,25 & 0,5 & 0 & 0,25 \\ 0,25 & -0,5 & 0 & 0,25 \\ 0,25 & 0 & 0,5 & -0,25 \\ 0,25 & 0 & -0,5 & -0,25 \end{bmatrix}, \tag{15}$$

which relates the vector of voltages at the inputs of the motors $\bar{u}$ with the vector of voltages $\bar{u}_r = [u_z, u_\theta, u_\phi, u_\psi]^T$ at the inputs of the compensator K_c (15), i.e. $\bar{u} = \mathrm{K}_c \bar{u}_r$.

In that case, the matrix block diagram of the control system can be represented in the form in Fig. 2, where k_{gi} are static gains [5, 7], and the matrix equation (13) takes the simple form

$$\bar{\eta} = W_U(s)\bar{u}_r + W_\gamma(s)\bar{\gamma} - \frac{1}{s^2} F g, \tag{16}$$

where, as stated above, the matrices $W_U(s)$ and $W_\gamma(s)$ are diagonal.

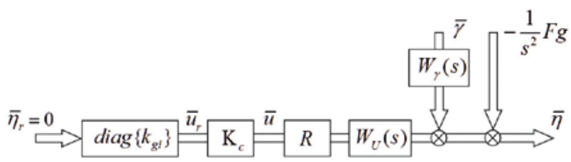


Fig. 2. Matrix block diagram of the control system with the compensator K_c (15)

Thus, the introduction of the compensator K_c (15) leads to the independence of the separate control channels of the quadcopter.

Let us consider, for example, the control channel by the pitch angle θ (the control of the other channels of the decoupled system is carried out similarly). Based on the equations (9) and (12) - (16) we can write

$$\theta = \frac{L}{I_x s^2}\frac{K_M \omega_M}{(s+\omega_M)}u_\vartheta + \frac{1}{I_x s^2}\gamma_\theta . \qquad (17)$$

The corresponding block diagram of the system is shown in Fig. 3.

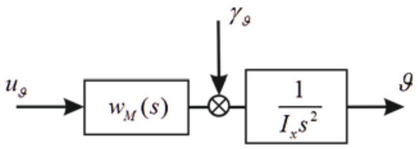


Fig. 3. Block diagram of the pitch cannel of the decoupled control system

The task is to develop an L_1 adaptive controller with a *state predictor* [5], which compensates for the arbitrary, but limited (i.e. $|\gamma_\theta| \le \Delta_0$) external disturbance γ_θ. Since the control signal of the system after entering the compensator K_c (15) is the voltage vector $\bar{u}_r$, let us transfer the disturbance γ_θ in (17) to the point of application of the voltage $u_\theta = \theta_r$. Based on the known rules of transformation of block diagrams [10], we can write:

$$\theta = \frac{L}{I_x s^2}\frac{K_M \omega_M}{(s+\omega_M)}\left(u_\vartheta + \gamma_E\right), \qquad (18)$$

where the equivalent disturbance γ_E is

$$\gamma_E = \frac{\gamma_\vartheta (s+\omega_M)}{L K_M \omega_M}. \qquad (19)$$

For developing an L_1 adaptive controller, we write the operator equation (18) in the state space form:

$$\frac{dx(t)}{dt} = Ax(t) + b\left[u_\vartheta(t) + \gamma_E(t)\right], \quad x(0) = x_0, \qquad (20)$$
$$y(t) = \vartheta(t) = cx(t),$$

where the three-dimensional state vector $x(t)$ has the following components: $\theta(t)$, $d\theta(t)/dt$, $d^2\theta(t)/dt^2$, and the constant matrix A and row- and column-vectors b and c can be expressed in the following form

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\omega_M \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 0 \\ L K_M \omega_M / I_x \end{bmatrix}, \qquad (21)$$
$$c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

The state predictor has the same structure as the system in (20):

$$\frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + b\left[u_\theta(t) + \hat{\gamma}_E(t)\right], \quad \hat{x}(0) = x_0, \qquad (22)$$
$$\hat{y}(t) = c\hat{x}(t),$$

and the only difference is that the unknown disturbance vector $\gamma_E(t)$ is replaced by its estimate $\hat{\gamma}_E(t)$.

The compensation of the disturbance $\gamma_E(t)$ is performed by the following adaptive control law [5, 11]:

$$\frac{d\hat{\gamma}_E(t)}{dt} = \Gamma b^T P \varepsilon(t) \qquad (23a)$$

or

$$\hat{\gamma}_E(t) = \Gamma b^T P \int \varepsilon(t) dt ,$$

(23b)

where $\varepsilon(t) = x(t) - \hat{x}(t)$ is the *prediction error*, P ($P = P^T > 0$) is the solution of the Lyapunov equation

$$A^T P + PA = -Q \qquad (24)$$

for an arbitrary symmetric positive definite function Q ($Q = Q^T > 0$). The positive scalar $\Gamma > 0$ in (23) is called the *adaptation gain* [5, 11].

According to L_1 adaptive control theory [5], the control signal $u_\theta(t)$ of the system is given in operator form as

$$u_\vartheta(s) = q(s)\left[k_g \vartheta_r(s) - \hat{\gamma}_E(s)\right], \qquad (25)$$

where $\theta_r(s) = 0$ is an one-dimensional reference signal, k_g is a static gain, and $q(s)$ is the transfer function of a low-pass filter, satisfying the DC gain condition $q(0) = 1$ [5].

The block diagram of the control system with the state predictor (22), the adaptive disturbance rejection law (23), and the control signal $u_\theta(s)$ (25) is shown in Fig. 4, where single lines correspond to scalar signals, and double lines correspond to vector ones.

As shown in [11], the output signal (error) $\vartheta_\gamma(s)$ of the adaptive system in Fig. 4 caused by the disturbance $\gamma_E(s)$ can be written in the operator form as

$$\vartheta_\gamma(s) = W(s)\left[1 - q(s)\left[I + W_0(s)\right]^{-1} W_0(s)\right]\gamma_E(s), \qquad (26)$$

where transfer functions $W(s)$ and $W_0(s)$ are

$$W(s) = c(sI - A)^{-1}b, \quad W_0(s) = \frac{\Gamma}{s}W_{PR}(s),$$
$$W_{PR}(s) = b^T P(sI - A)^{-1}b . \qquad (27)$$

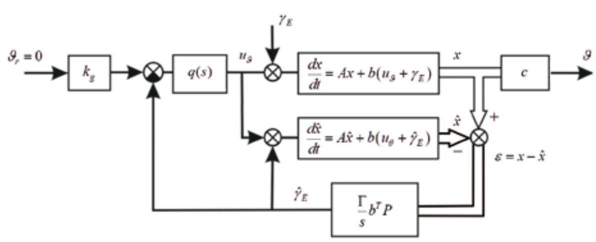


Fig 4. Block diagram of the L_1 adaptive control system with the state predictor

Note that $W(s)$ in (26) and (27) is the transfer function of the quadcopter open-loop control system of the pitch channel after entering the matrix decoupling compensator K_c (15), and $W_{PR}(s)$ in (27) belongs to the class of *Positive Real* transfer functions, the phase shift of which does not exceed -90° [6, 11].

The equation (26) describes a system, the block diagram of which is shown in Fig. 5, where the transfer function $F_{\Sigma}(s)$ in the negative feedback loop is given by the expression

$$F_{\Sigma}(s) = \left[I + W_0(s)\right]^{-1} W_0(s)\,. \qquad (28)$$

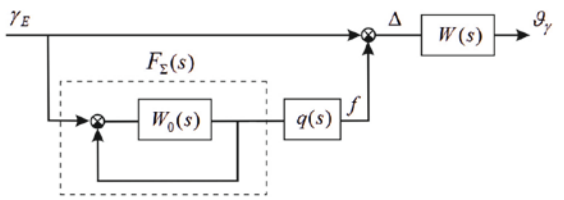


Fig. 5. Equivalent block diagram of the adaptive system with respect to the disturbance $\gamma_E(s)$

Since the phase shift of the transfer function $W_{PR}(s)$ in (27) does not exceed -90°, and the integrator in $W_0(s)$ introduces a constant phase shift -90°, it is obvious that the system with the transfer function $F_{\Sigma}(s)$ (28) is stable for any value of the adaptation coefficient Γ [since the locus $W_0(j\omega)$ in the complex plane cannot encircle the critical $(-1, \mathrm{j}0)$ point for any values of $\Gamma > 0$ and $\omega > 0$].

Moreover, if we assume for simplicity that $q(s) = 1$, then from (27) and Fig. 5, it can be seen that for $\Gamma \to \infty$ we have $F_{\Sigma}(s) \to 1$, and the error ϑ_γ tends to zero regardless of the magnitude and form of the disturbance $\gamma_E(t)$. In other words, by choosing the magnitude of the adaptation gain Γ we can reduce the error $|\vartheta_\gamma|$ to any arbitrarily small quantity.

4. CONCLUSION

The task of developing an adaptive control system for compensating external disturbances (for example, wind gusts) acting on quadcopter is considered, under the assumption that there are no translational motions of the quadcopter in the inertial space. The proposed system can be used in the development of fault-tolerant control systems for multi-rotor UAVs.

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