CSIT Conference 2021, Yerevan, Armenia, September 27 - October 1

On One Approach to Computer Modeling of the Dynamics of Heat and Power Systems by the Volterra Series Method

Ekaterina Antipina
ESI SB RAS, ISU
Irkutsk, Russia
e-mail: kate19961231@gmail.com

Evgeniia Markova
ESI SB RAS
Irkutsk, Russia
e-mail: markova@isem.irk.ru

Svetlana Solodusha
ESI SB RAS
Irkutsk, Russia
e-mail: solodusha@isem.irk.ru

***Abstract*—The paper focuses on the construction of information models for the technical energy systems' functioning. This is an urgent problem that arises in applications when describing the dynamics of systems of the "input-output" type. Constructive algorithms for solving typical inverse problems arising when evaluating the reliability of power systems are implemented. The scientific novelty of the work is due to the use of new methods based on nonclassical Volterra integral equations of the first kind as applied to the identification and automatic control problems in describing the nonlinear dynamics of technical equipment of a large electric power system. The results of a computational experiment are presented. They confirm the accuracy of the developed computational algorithms, sufficient for practical application.**

***Keywords*—Volterra series, modeling of nonlinear dynamical systems, heat power engineering.**

I. INTRODUCTION

At present, there is a wide range of tools for the study of nonlinear dynamic systems. Several types of software can be distinguished, depending on the level of formalization of the problems considered by the researcher. As noted in [1], the most popular are packages of universal systems based on the visual principle of modeling, for example, MatLab Simulink, MvStudium. This is due to the successful combination of the advantages of computer mathematics systems, in which the program code is formed using symbolic mathematical records, and systems for modeling highly specialized processes in a specific area of knowledge, in which the computer model is implemented using the corresponding universal basic components.

The analysis of scientific and technical literature shows the relevance of software development for modeling dynamic systems using the Volterra integro-power series [2]. The VoltaireXL program of the Microwave Office 2000 system is well known. It uses the Volterra series theory to analyze the transient modes of electronic circuits in the frequency domain [3]. However, no universal software has yet been created for the application of this mathematical apparatus in the time domain [4]. It seems surprising that there are no mathematical modeling tools based on Volterra polynomials in the libraries of modern programming systems. This is largely due to the complexity of the problem of identifying Volterra kernels. Therefore, researchers create authoring software written in a high-level programming language, in particular C++, Pascal, Fortran. Although this approach is quite laborious, it nevertheless allows the implementation of universal modules that will be in demand when describing various technical systems. In this regard, packages are being developed (see, for example, [1,5-7]), the algorithmic implementation of which is based on the author's techniques.

This paper describes an approach to modeling the nonlinear dynamics of heat-and-power objects using the theory of non-classical Volterra integral equations of the first kind [8, 9], the application of which allows one to construct effective computational algorithms in real time. This work continues the research that began in [10-12]. The text of the paper is organized as follows: Section 2 contains information on the dynamic system and simulation models for describing the nonlinear dynamics of thermal power equipment with vector input signals. Section 3 describes the experimental procedure for the formation of a training sample, based on which the identification of Volterra kernels is performed, the methodology for constructing integral models based on the quadratic Volterra polynomials, and the results of computer simulation. Section 4 gives a perspective on future workings. Finally, Section 5 discusses the main results.

II. SYSTEM DINAMICS AND SIMULATION MODELS

The traditional modeling approach involves the use of differential equations based on information about the physical nature of a dynamic object. In particular, in the case of a linear dynamic system, the input signals $x(\tau)$ of which consist of control $x_1(\tau)$ and external $x_2(\tau)$ influences, the simulation model can be represented as follows:

$$\dot{z}(\tau) = A(\tau)z(\tau) + B(\tau)x_1(\tau) + x_2(\tau), \tau \in [\tau_0, T], \quad (1)$$

$$y(\tau) = G(\tau)z(\tau), \quad (2)$$

where τ is the time, $z(\tau) \in R^n$ is the phase vector, $x_1(\tau)$ is the r-dimensional vector of control input, $x_2(\tau)$ is the n-dimensional vector of measured external signals, $A(\tau)$, $B(\tau)$, and $G(\tau)$ represent, respectively, the square $(n \times n)$ and rectangular $(n \times r)$, $(m \times n)$ matrices of parameters, $y(\tau)$ is the m-dimensional vector output. A model of the form (1), (2) can be transformed into the "input-output" type model. Indeed, assuming that all elements $A(\tau)$, $B(\tau)$, $G(\tau)$, $x_1(\tau)$, and $x_2(\tau)$ belong to the space of real continuous functions, we write the solution to (1) in the form

$$z(\tau) = Z[\tau, \tau_0]z(\tau_0) + \int_{\tau_0}^{\tau} H[\tau, s]x_1(s)ds + \int_{\tau_0}^{\tau} X[\tau, s]x_2(s)ds, \quad (3)$$

where $H[\tau, s] = X[\tau, s]B(s)$, and $X[\tau, s]$ denotes the normalized fundamental matrix of the homogeneous part of the equation (1). For $\tau_0 = 0$, $x(\tau_0) = 0$, from (2), (3), we obtain the Volterra integral equation of the first kind

$$y(\tau) = \int_0^{\tau} K_1(\tau, s)x_1(s)ds + \int_0^{\tau} K_2(\tau, s)x_2(s)ds, \; \tau \in [0, T], \quad (4)$$

where $K_1(\tau, s) = G(s)H[\tau, s]$, $K_2(\tau, s) = G(s)X[\tau, s]$ are interpreted as impulse-transient functions. A model of the form (4) is used when there is no a priori information on the (physical) structure of the dynamic system. In the case of a nonlinear stationary dynamical system (the transient characteristics of which do not explicitly depend on time), instead of (4), we have a model in the form of a finite segment of the N-th degree (polynomial) of the Volterra functional series

$$y(\tau) = \sum_{m=1}^{N} \int_0^{\tau} \ldots \int_0^{\tau} K_{\underbrace{1\ldots1}_{m}}(s_1, \ldots, s_m) \prod_{i=1}^{m} x_1(\tau - s_i)ds_i +$$
$$+ \int_0^{\tau} \ldots \int_0^{\tau} K_{\underbrace{2\ldots2}_{m}}(s_1, \ldots, s_m) \prod_{i=1}^{m} x_2(\tau - s_i)ds_i + \quad (5)$$
$$+ \sum_{p+q=m} \int_0^{\tau} \ldots \int_0^{\tau} K_{\underbrace{1..1}_{p}\underbrace{2..2}_{q}}(s_1, \ldots, s_m) \prod_{i=1}^{p} x_1(\tau - s_i) \prod_{j=1}^{q} x_2(\tau - s_j)ds_i ds_j,$$

where $\tau \in [0, T]$. This paper considers the nonlinear dynamics of the local section of the steam-water path of the power unit of Nazarovo power plant, which includes an 80-KTC-1 condenser and a low-pressure heater. A change in the cooling water flow $x_1(\tau) \equiv \Delta D_w(\tau)$ and a change in the steam flow $x_2(\tau) \equiv \Delta D_s(\tau)$ passing through the condenser are selected as input signals. The output signals are the pressure deviation Δp in the condenser, deviation of the water temperature Δt_1 at the outlet of the condenser, and Δt_2 at the outlet of the low-pressure heater. The initial values of the input and output signals are $p_0 = 4359$ Pa, $t_{1_0} = 15.20$ ºC, $t_{2_0} = 59.19$ ºC, $D_{w_0} = 11562.2$ kg/s, and $D_{s_0} = 51.46$ kg/s, correspondingly. Figure 1 shows the block diagram of the simulated dynamic systems. Designations **IM-I**, **IM-II**, **IM-III** denote blocks for constructing integral models based on Volterra polynomials (5) at $N = 2$ and the input signal $\Delta D(\tau) = (\Delta D_w(\tau), \Delta D_s(\tau))^T$. The block diagram of the simulation model (**SM**) contains data sets $(\Delta D(\tau), \Delta p(\tau))$, $(\Delta D(\tau), \Delta t_1(\tau))$, $(\Delta D(\tau), \Delta t_2(\tau))$ for the selected section of the steam-water path, required for the construction and verification of integral models. The simulation model is a development of the model of the power unit of the Irkutsk Irkutsk Central Heating and Power Plant (CHPP-10) [13] and includes about one hundred algebraic-differential and five hundred algebraic equations with closing relations.

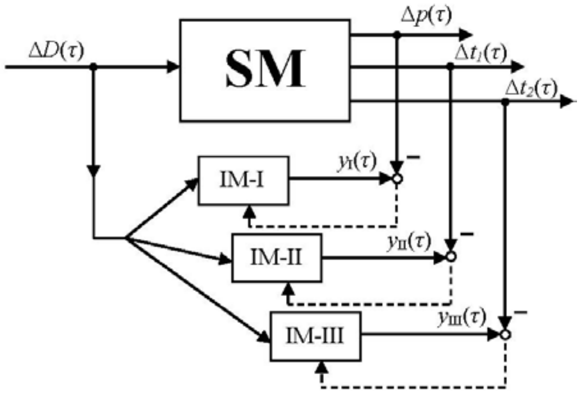

Fig. 1. Block diagram of dynamic systems simulation.

III. Methodology and Results

A. Motivation

Let us briefly consider some software packages for modeling dynamic systems using the Volterra series method in the time domain. Basically, they are aimed at solving the problem of identifying the transient characteristics of a specific dynamic object to monitor the response to arbitrary test signals. In particular, in [14], a description of the "Tools of Identification Nonlinear Dynamic Objects" package is given. It consists of blocks for modeling, identification, setting the initial parameters, and filtering external noise in the data. The instrumental environment for identifying nonlinear dynamic objects was developed in MatLab Simulink using Volterra polynomials of the second and third degrees [15]. There, a software tool for parallel computing is implemented in the Java language to improve performance. These software systems were tested for indirect diagnostics of the air gap between the rotor and the stator of a reluctance motor [16,17]. Diagnostic algorithms are based on the results of identifying the diagonal values of Volterra kernels characterizing the technical object dynamics.

Based on the analysis of the functional capabilities of existing software tools and scientific and technical literature on the use of the Volterra series tool, it is possible to formulate some promising research areas in which the use of computer modeling tools is effective:

- test diagnostics of the technical condition of the object,
- monitoring the object's response to input influences in the investigated range of values,
- automatic control of nonlinear dynamics of an object to correct input disturbances.

In this paper, we apply a modeling technique based on Volterra polynomials. It consists of the stage of solving the identification problem and building the information model of the object and the stage of identifying the input actions, which arises in the problem of automatic control of technical objects.

B. Data Generation

To construct three quadratic Volterra polynomials of the form (5) at $N = 2$ with responses $y_I(\tau) \equiv \Delta p(\tau)$, $y_{II}(\tau) \equiv \Delta t_1(\tau)$,

$y_{III}(\tau) \equiv \Delta t_2(\tau)$, describing the response of the dynamic systems **IM-I**, **IM-II**, **IM-III** from Figure 1, we carry out two series of tests of the form

$$x_{l_\omega}(\tau) = \alpha_{l,m}\left(e(\tau) - e(\tau - \omega)\right),\ 0 \le \omega \le \tau \le T,\ m = 1, 2, \quad (6)$$

where $e(\tau)$ is the Heaviside function, $\alpha_{l,1} = -\alpha_{l,2}$ are test signal amplitudes. Each of the series of signals (6) is aimed at identifying symmetric Volterra kernels K_I, K_{II}, $l = 1, 2$. At the same time, amplitudes of $\alpha_{1,1} = -\alpha_{1,2} = 25\% D_{w_0}$ are chosen for test actions $\Delta D_w(\tau)$, and $\alpha_{2,1} = -\alpha_{2,2} = 10\% D_{s_0}$ are used for actions $\Delta D_s(\tau)$.

Difference analogs of symmetric Volterra kernels are obtained on a uniform grid $\tau_i = ih$, $\omega_j = jh$, $i = \overline{1,n}$, $j = \overline{1,i}$, $T = nh$. The time range is determined based on the results of evaluative testing of dynamic systems and was $T = 120$ s. Figures 2, 3 illustrate responses $\Delta p(\tau, \omega)$ to inputs $\Delta D_w(\tau)$ of type (6) for $h = 8$ s, used to identify K_{11}. The graphs in the figures are numbered according to the durations ω_j, $j = \overline{1,15}$.

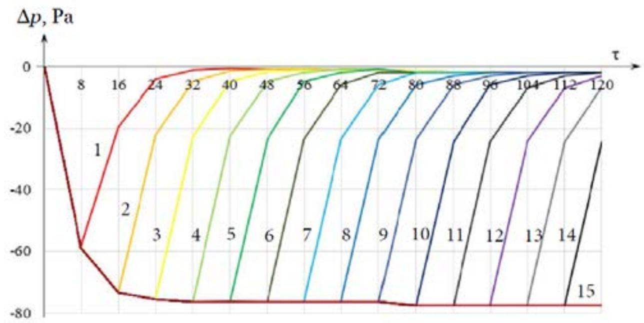

Fig. 2. Responses $\Delta p(\tau, \omega)$ to inputs $\Delta D_w(\tau)$ at $\alpha_{1,1} = 25\% D_{w_0}$.

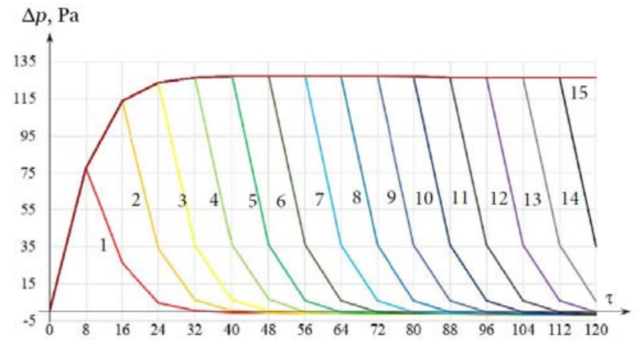

Fig. 3. Responses $\Delta p(\tau, \omega)$ to inputs $\Delta D_w(\tau)$ at $\alpha_{1,2} = -25\% D_{w_0}$.

Additional series of tests are carried out to identify nonsymmetric kernels from (5):

$$\begin{cases} x_{1_\omega}(\tau) = \alpha_{1,1}\left(e(\tau) - e(\tau - \omega)\right), \\ x_2(\tau) = \alpha_{2,1} e(\tau), \end{cases}$$

$$\begin{cases} x_1(\tau) = \alpha_{2,1} e(\tau), \\ x_{2_\omega}(\tau) = \alpha_{1,1}\left(e(\tau) - e(\tau - \omega)\right), \end{cases}$$

A detailed description of these tests is given in [12]. Note that the procedure for processing output signals using cubic splines [18] proved itself well in the case of noisy input data.

C. Results

In addition to the identification stage, the scheme includes a stage for comparing the quality of models, noted in Fig. 1 with a dotted line. This stage is implemented on signals from the feasible set of input actions that do not participate in the identification procedure. Testing the effectiveness of integral models shows the modeling accuracy sufficient for practical use. In particular, on a series of test signals for $\Delta D_s(\tau) \equiv 0$ and

$$\Delta D_w^{(1)}(\tau) = 25\% D_{w_0}\left(e(\tau) - 2e(\tau - 32) + e(\tau - 120)\right),$$

$$\Delta D_w^{(2)}(\tau) = 30\% D_{w_0}\left(e(\tau) - 2e(\tau - 64) + e(\tau - 120)\right),$$

we obtain

$$\varepsilon_I^{(1)}(\tau) = 1.756\ (0.040\% p_0),\ \ \varepsilon_I^{(2)}(\tau) = 8.056\ (0.185\% p_0),$$

$$\varepsilon_{II}^{(1)}(\tau) = 0.002\ (0.012\% t_{1_0}),\ \ \varepsilon_{II}^{(2)}(\tau) = 0.041\ (0.272\% t_{1_0}),$$

$$\varepsilon_{III}^{(1)}(\tau) = 0.004\ (0.067\% t_{2_0}),\ \ \varepsilon_{III}^{(2)}(\tau) = 0.070\ (0.118\% t_{2_0}).$$

Note that the algorithms in software modules for identifying Volterra kernels are universal and were tested when simulating the nonlinear dynamics of various heat and electric power objects [19].

IV. Future Work

The implementation of the stage of restoring the input actions in (5), providing the desired response of the dynamical system, relies heavily on upper exact estimates for the right boundary T of the existence domain of a (unique) continuous real solution to the Volterra polynomial equations of the first kind [9]. We plan to develop a series of special test integral equations of the first kind, in which the Volterra kernels satisfy the specified conditions, to evaluate the possibilities of numerical methods for solving them. For this, a constructive algorithm has been developed for constructing multidimensional functions majorizing the impulse-transient functions of a dynamic object [20]. Figures 4, 5 show the qualitative behavior of the Volterra kernels from (5) marked in red and the corresponding majorants marked in blue.

We plan to use the developed procedure for the approximation of Volterra kernels to carry out a multivariate analysis of the domains of determining the control input signals.

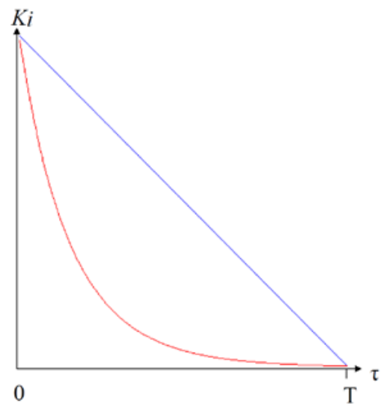

Fig. 4. Kernel $K_i(\tau)$ and its majorant.

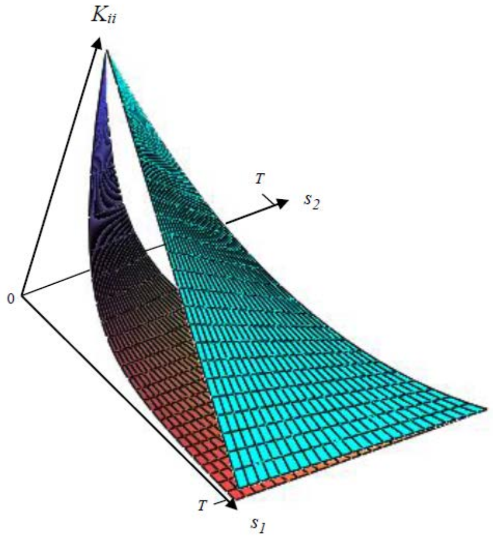

Fig. 4. Kernel $K_{ii}(s_1, s_2)$ at $s_1 \geq s_2$ and its majorizing surface.

V. CONCLUSION

The paper presents a methodology based on the method of the Volterra series for constructing information models for the power systems' functioning. The author's algorithms related to the simulation of the automatic control system of technical objects are used to describe the nonlinear dynamics of equipment at the local section of the Nazarovo power plant. The constructive schemes of the implemented algorithms are based on numerical methods for solving nonclassical Volterra integral equations of the first kind. The results of the computational experiment confirm the effectiveness of the proposed approach.

ACKNOWLEDGMENT

The research was carried out within the state assignment of Ministry of Science and Higher Education of the Russian Federation (Project FWEU-2021-0006, theme № AAAA-A21-121012090034-3).

Researcher Antipina E.D. was partially supported by the grant NIR-2021 for graduate students and young employees of ISU (theme № 091-21-301).

The results of the computational experiment were obtained using the resources of the High-Temperature Circuit Multi-Access Research Center (Project № 13.ЦКП.21.0038).

REFERENCES

[1] O.R. Volkova, P.V. Reshetnikov, "Intelligent operational diagnosis, identification and modelling of technical systems based on time series", *Bulletin of MSTU "Stankin"*, vol. 1(19), pp. 100-103, 2012.

[2] V. Volterra, *The theory of functionals, integral and integro-differential equations*, Nauka, Moscow, 1982.

[3] C.M. Cheng, Z.K. Peng, W.M. Zhang, G. Meng, "Volterra-series-based nonlinear system modeling and its engineering applications: A state-of-the-art review", *Mechanical Systems and Signal Processing*, vol. 87, pp. 340-364, 2017.

[4] *Applied Wave Research*, E. Grand Ave., El Segundo, California, 90245, USA, 1960.

[5] V.M. Glushan', L.A. Zinchenko, "Software support for CAD, modeling and synthesis of electronic circuits based on differential functional polynomials", *News of SFedU. Engineering Sciences*, vol. 2, no. 8, pp. 93-97, 1998.

[6] V.A. Speransky, "Instrumental tools for constructing models of nonlinear systems in the form of Volterra series in the frequency domain", *Bulletin of the National Technical University of KhPI*, vol. 68, no. 974, pp. 110-115, 2012.

[7] T.Yu. Tsibizova, "Identification of Nonlinear Systems of Automatic Control by Means of Voltaire's Filters", *Basic Researches*, vol. 2, no. 14, pp. 3070-3074, 2015.

[8] A.S. Apartsyn, *Nonclassical linear Volterra equations of the first kind*, VSP Utrecht, Boston, 2003.

[9] S.V. Solodusha, "Methods for constructing integral models of dynamic systems: algorithms and applications in power engineering", Doctoral PhD thesis, ESI SB RAS, Irkutsk, Russia, 2019.

[10] E.D. Antipina, V.A. Spiryaev, E.A. Tairov, "Application of a Volterra quadratic polynomial to modeling elements of heat engineering devices", *E3S Web of Conferences*, vol. 114, pp. 01007, 2019.

[11] S.V. Solodusha, V.A. Spiryaev, E.A. Tairov, "Numerical Modeling of Dynamics of Thermal Power Equipment of the Power unit at the Nazarovo Power Station by Volterra Polynomial", *2019 XXI International Conference Complex Systems: Control and Modeling Problems*, Samara, Russia, 03-06 September 2019, IEEE, pp. 616-619, 2019.

[12] E.D. Antipina, E.A. Tairov, E.V. Markova, "Studying the complexity of identification of Volterra kernels for the case of a vector input signal of arbitrary dimension", *E3S Web of Conferences*, vol. 209, pp. 03004, 2021.

[13] E.A. Tairov, A.A. Loginov, E.A. Chistyakov, *Mathematical model, numerical methods and software of the simulator for the generating unit of the Irkutsk CHP-10*, Irkutsk, SEI SB RAS, Preprint no. 11, 1999.

[14] M.M. Masri, "Methods and tools for constructing information models of nonlinear dynamic objects for diagnostic purposes", PhD thesis, Odessa Polytechnic University, Odessa, Ukraine, 2013.

[15] S.V. Pavlenko, "Methods and tools for identification nonlinear dynamical systems with using Volterra models", PhD thesis, Odessa Polytechnic University, Odessa, Ukraine, 2017.

[16] V.D. Pavlenko, "Information technology of indirect control and diagnostics of dynamic objects based on Volterra models", *Proceedings of Odessa Polytechnic University*, vol. 2, no. 30, pp.194-199, 2008.

[17] S.V. Pavlenko, V.D. Pavlenko, "Building an information model of the photosynthetic reaction center in the form of Volterra kernels", *Proceedings of the XI Intern. Sci.-Tech. Conf. "Analytical and numerical methods for modeling natural scientific and social problems"*, Penza, PSU Publishing House, pp. 100-104, 2016.

[18] Yu.E. Voskoboinikov, V.A. Boeva, "Stable algorithm for nonparametric identification in the presence of anomalous measurements", *Computational Technologies*, vol. 25, no. 3, pp. 46-53, 2020.

[19] S.V. Solodusha, "Amplitudes of Test Signals for Identification of Volterra Kernels", *2018 International Conference on Industrial Engineering, Applications and Manufacturing*, Moscow, Russia, May 15-18 2018, IEEE, pp. 8728575, 2018.

[20] S.V. Solodusha, E.Yu. Grazhdantseva, "Test polynomial Volterra equation of the first kind in the problem of identifying input signals", *Proceedings of the IMM UB RAS*, vol. 4, 2021 (in print).