Network Infrastructures Assessment Stability

Timur V. Jamgharyan
National Polytechnic University of Armenia
Yerevan, Armenia
e-mail: t.jamgharyan@yandex.ru

Vahe H. Ispiryan
National Polytechnic University of Armenia
Yerevan, Armenia
e-mail: vaheispiryan4@gmail.com

***Abstract* — The article proposes to consider the choice of parameters of the mathematical model of the network for its optimization. The analysis of the network behavior is estimated using the Lyapunov stability theory. This can serve as a rough estimate of the state of the networked system.**

Keywords—multicriteria model, stratification, Lyapunov stability, Pareto optimal, network infrastructure, continuous time.

I. Introduction

When deploying and operating a network infrastructure, there are often situations in which the decision to change one or another element included in the network can lead to a violation of stability and is not optimal.

Network management is carried out on the basis of a set of data entered into the devices in accordance with the protocols and parameters of all components the network infrastructure.

This issue is especially sensitive at the core layer of the network because, unlike the access and distribution layers, instability at the core layer affects the entire network infrastructure [1]-[2]-[3].

It is also necessary to consider the impact of local multi-criteria changes on the entire global network infrastructure. At this stage the issue of network infrastructure as a rule boils down to an increase in the transmission speed and volume of data, and the issue of sustainability is determined by the reliability of the software and hardware components of the network, where the influence of an external destabilizing factor is minimal.

The issues of choosing true criteria from a multicriteria sample of the flow of event [4]-[5] are becoming relevant, with a quick reconfiguration of the network infrastructure in a limited time.

The purpose of the article is to develop a model for assessing the resilience of a network infrastructure during rapid reconfiguration. To achieve this purpose the following tasks are considered and solved in the article: the concept of the stability function of the network infrastructure is introduced, a network model is developed in the form of a multicriteria dataset, and an algorithm for assessing the stability is developed.

There are many situations in which and in relation to which it is required to make a choice. As a rule, the number of single-criterion elections is few, mainly the number of elections is multicriteria. Today, there is a wide range of solutions to solve this problem: from heuristic to axiomatic. However, it should be recognized that there is no one "ideal" method [6]-[7]. The problem of abstract choice consists in specifying one or more options from the available initial set of possible (feasible) options (solutions) that are call selectable.

It should also be noted that with an increase in the amount of and the speed of data transmission over the network (especially at the network core layer), the reliability requirements for devices and software environments operating at different layers of the OSI model are growing [8]-[9]-[10]-[11].

In a network infrastructure, a change in one parameter (configuration file) can lead to unstable operation of the entire infrastructure after a long time from the moment of the change.

The systematic approach to calculating the stability of the network infrastructure also allows you to assess the degree of fault tolerance.

Along with the structural characteristics of the network, it is necessary to take into account many local factors (compatibility of software versions, protocols level of training of personnel in a geographically distributed network infrastructure) [12].

The application of the Pareto principle also enables the formulation of scientific and statistical policy for the network infrastructure [13]-[14].

Based on the foregoing, an urgent problem in the design and operation of network infrastructures is the task of building a model for assessing the stability of a network infrastructure in the context of its multicriteria optimization [4].

II. Terms and Definition

A. Lyapunov stable theory basic

Equation solution $\frac{dx_i}{dt} = f_i(x_1, x_2, \dots\dots, x_m, t)$ (1)

where $\varphi_i(t)$, $i = 1,2, \dots\dots, m$ satisfying the initial conditions $\varphi_i(t_0) = \varphi_{io,}$ $i = 1,2, \dots\dots, m$ is **called Lyapunov stable**, for $t \longrightarrow \infty$ if for any $\varepsilon > 0$ there is such $\delta(\varepsilon) > 0$, for every solution $x_i(t), i = 1,2, \dots\dots m$, equations (1) initial values satisfy the conditions

$$|x_i(t_0) - \varphi_{io}| < \delta, \ i = 1,2, \dots\dots, m \quad (2)$$

$$|x_i(t) - \varphi_i| < \varepsilon, \quad i = 1,2, \dots\dots, m \quad (3)$$

For all $t \geq t_{0,}$ where t_0 starting point [15]-[16].

- if for arbitrarily small $\delta > 0$ for at least one solution $x_i(t), i = 1,2, \dots\dots m$ inequalities (3) are not satisfied, then the solution $\varphi_i(t)$ is **called unstable**,
- if in addition to the implement of inequalities (3) under condition (2), the condition

$$\lim_{x\to\infty} |x_i(t) - \varphi_i(t)| = 0 \quad (4)$$

where: $i = 1,2, \dots\dots, m$ then the solution $\varphi_i(t)$ $i = 1,2, \dots\dots, m$ **is called asymptotically stable** [17].

B. The Pareto principle

Pareto rule (20/80 principle) means that 20% of the efforts give 80% of the result, and the remaining 80% effort only 20% of the result [18]-[19].

A Pareto chart is a type of chart that contains columns and a line graph, where individual values represented by columns in descending by order, and the cumulative total is represented by a line [20].

A decision maker is a person who makes a choice and bears full responsibility for its consequences [4].

C. Network technology

Network infrastructure is a collection of special equipment and software that creates the basis for effective information exchange [21].

An Autonomous System (AS) is a system of IP networks and routers by managed one or more operators with a uniform routing policy with the Internet [22] - [23].

Autonomous system number (ASN) is a unique identifier of a set of communication facilities organized on the System, with uniform control policies [23].

III. Model Parameters

To develop a mathematical model of a stratified network infrastructure, introduce the following notation and apply the OSI (Open Systems Interconnection) model [11]-[24].

1. R — many criteria of system elements

$$\left\{r_i^{(1)}, r_i^{(2)}, \dots\dots, r_i^{(n)}\right\} \in R \qquad (5)$$

where:

$r_i^{(1)}$— subset of physical layer devices criteria,

$r_i^{(2)}$— subset of data link layer devices criteria,

$r_i^{(3)}$— subset of network layer devices criteria,

$r_i^{(4)}$— subset of transport layer devices criteria,

$r_i^{(n)}$—subset of criteria for other devices (uninterruptible power supplies, antenna mast devices, antenna feeder devices).

2. D —many declared system criteria (parameters, protocols, algorithms of functioning)

$$\left\{d_i^{(1)}, d_i^{(2)}, \dots\dots, d_i^{(n)}\right\} \in D \qquad (6)$$

where:

$d_i^{(1)}$ —subset of static system criteria (trunk equipment, communication lines),

$d_i^{(2)}$ —subset of dynamic system criteria,

$d_i^{(3)}$ —subset of the degrees of freedom of the dynamic criteria of the system (range of IP addresses, range of allowed protocol changes, number of traffic encapsulations),

$d_i^{(4)}$—security policy criteria subset system [25]-[26],

$d_i^{(n)}$—subset of other system criteria (by the subset of other system criteria we mean possible but not applicable subsets of the system criteria (for example, the network can use IPv4 and IPv6 protocols, but only IPv4 is used, or a selected set of encryption and authentication algorithms out of all possible for the IPsec protocol)).

3. G — many criteria for undefined system states

$$\left\{g_i^{(1)},\ g_i^{(2)}, \dots\dots, g_i^{(n)}\right\} \in G \qquad (7)$$

where:

$g_i^{(1)}$—subset of criteria for uncertainty due to propagation medium,

$g_i^{(2)}$—subset of criteria for uncertainties arising from system vulnerabilities (undeclared capabilities of hardware and software that can be additional attack vectors),

$g_i^{(3)}$—subset of criteria for uncertainties associated with an incomplete description of the system,

$g_i^{(4)}$ —subset of criteria for uncertainties associated with statistical errors of processes occurring in devices and software environments used in network infrastructure (thermal noise, software errors),

$g_i^{(n)}$ —subset of other system criteria (by the subset of other criteria of the system we mean various known but not fixed vulnerabilities, the presence of insiders among the staff, the human factor itself in the matter of configuring the network infrastructure, ASN).

Due to the high level of abstraction, the G set includes both parametric and structural uncertainties [27].

4. T —network (AS) design topologies

$$\left\{t_i^{(1)}, t_i^{(2)}\right\} \in T \qquad (8)$$

where:

$t_i^{(1)}$— fully connected AS topology,

$t_i^{(2)}$— hybrid connected AS topology.

Network infrastructure built on the base of complex mutually complementary construction topologies.

Fully connected topologies in large networks are rarely used, since *N* communications required for communication of *N*(N-1)/2* nodes, that is, there is a quadratic dependence on the number of nodes [28].

5. Q (stability test function) —criterion which determines the set of all states of the system (the multicriteria of the system) within a given measurement at continuous (within the set) time.

The Q criterion for each dataset is specified by a function that is independent of the system and is determined by external factors (for example, for $g^{(1)}$ — the noise function, for $g^{(2)}$ —undeclared software and hardware capabilities, $d^{(2)}$ the update release time, or software support time).

Since the function Q is independent of the system, in some cases it can become an external destabilizing factor.When forming a stratified multicriteria network infrastructure model, it is necessary to use the stratification accuracy indicator.

Based on [27], we obtain the formulas for the accuracy of the network infrastructure stratification.

The calculations of multicriteria stratification presented below were performed under the assumption of mutual statistical independence of the data. r_i, z_i, g_i, t_i.

$$K(R) = \sum_{i=1}^{n_1} \frac{K_i(r_i)}{n_1} \quad (9)$$

where: $K(R)$ — accuracy of stratification of the set of criteria for system elements,

$$K(D) = \sum_{i=1}^{n_2} \frac{K_i(d_i)}{n_2} \quad (10)$$

where: $K(D)$ — the accuracy of stratification of the declared system criteria,

$$K(G) = \sum_{i=1}^{n_3} \frac{K_i(g_i)}{n_3} \quad (11)$$

where: $K(G)$ — the accuracy of stratification of the declared system criteria,

$$K(T) = \sum_{i=1}^{n_4} \frac{K_i(t_i)}{n_4} \quad (12)$$

where: $K(T)$ — system topologies stratification accuracy

where:

$K_i(r_i)$, $K_i(d_i)$, $K_i(g_i)$, $K_i(t_i)$— types of subsets to be stratified

n_1, n_2, n_3, n_4— number of objects of subsets to be stratified

M—unstratified network infrastructure

$K(M)$—stratified network infrastructure.

$$(R, D, G, T, Q) \in M \quad (12)$$

$$(K(R), K(D), K(G), K(T), Q) \in K(M) \quad (13)$$

It should be noted that in the practical calculation of the stability of the network infrastructure, it is not always necessary to carry out an accurate calculation of all stratified sets included in formula 13.

IV. Analysis Scheme

1. Determination of criteria for evaluating the network infrastructure of (R, D, G, T, Q) dataset.
2. Stratification of a dataset of criteria $K(R), K(D), K(G), K(T)$ and their subsets,
3. Determination of the necessary criterion for assessing and impact on the network infrastructure from the datasets criteria.
4. Obtaining the Q criterion (stability test function),
5. Determination by the Pareto method of the minimum set of criteria (S) allowing the optimal result within the framework of multicriteria choice,
6. Comparison S values with Q value
   - $S > Q$ network infrastructure is stable
   - $S < Q$ network infrastructure is not stable
   - $S = Q$ network infrastructure is stable asymptotically.

The $S = Q$ variant is possible for static (within continuous time) network infrastructures (that is, the network configuration and the set of elements when do not change or change very slowly).

V. Implementation

An example of calculating the resilience of a network infrastructure with equal numerical values of datasets with criteria and different resilience functions.

- $r_1^{(1)}, r_1^{(2)}, r_1^{(3)}, r_1^{(4)} = r_2^{(1)}, r_2^{(2)}, r_2^{(3)}, r_2^{(4)}$
- $d_1^{(1)}, d_1^{(2)}, d_1^{(3)}, d_1^{(4)} = d_2^{(1)}, d_2^{(2)}, d_2^{(3)}, d_2^{(4)}$
- $g_1^{(1)}, g_1^{(2)}, g_1^{(3)}, g_1^{(4)} = g_2^{(1)}, g_2^{(2)}, g_2^{(3)}, g_2^{(4)}$
- $K_1(r, d, g)_{1-4} = K_2(r, d, g)_{1-4}$
- $Q_1 \neq Q_2$
- the projected network of a heterogeneous type
- the entered datasets were obtained during the design of the network infrastructure of the distribution layer
- Statistical calculations were in the IBM SPSS Statistics and IBM SPSS Modeler software environment [29].

The "Pareto optimal" distribution in the network infrastructure of the elements of the datasets *R, D, G* presented in Tables 1, 2, 3 and Figures 1, 2, 3.

TABLE 1

"R" Datasets For Network Infrastructure

$r_{1,2}^1$	$r_{1,2}^2$	$r_{1,2}^2$	$r_{1,2}^4$
27	82	46	62

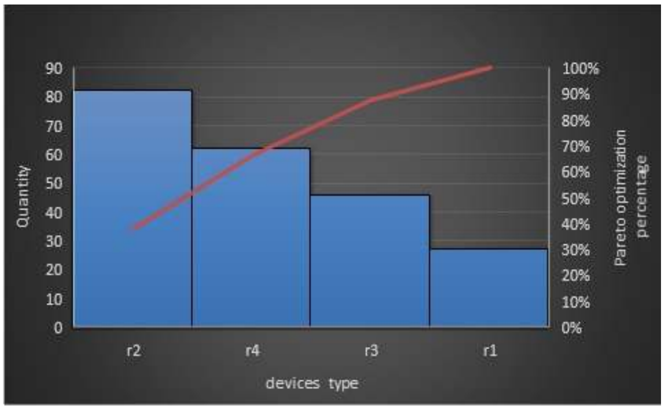

Fig. 1. The "Pareto optimal" distribution of elements of the dataset *r* in the network infrastructure

TABLE 2

"D" Datasets For Network Infrastructure

$d_{1,2}^1$	$d_{1,2}^2$	$d_{1,2}^3$	$d_{1,2}^4$
82	23	98	102

The values of criteria $d_{1,2}^2$and $d_{1,2}^3$ given in Table 2 defined as:

1. $d^2_{1,2}$—dataset used protocols
   - set of routing protocols used [30] - [31]
   - encryption and authentication protocols (in particular the basic IPsec set) [32]
   - network infrastructure stability monitoring software.
2. $d^3_{1,2}$—dynamic address change systems use only IPv4.

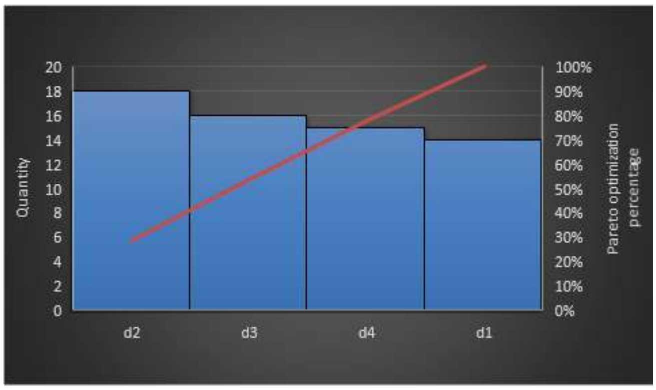

Fig. 2. The "Pareto optimal" distribution of elements of the dataset *d* in the network infrastructure

TABLE 3

"D" Datasets For Network Infrastructure

$g^1_{1,2}$	$g^2_{1,2}$	$g^3_{1,2}$	$g^4_{1,2}$
5	32	8	18

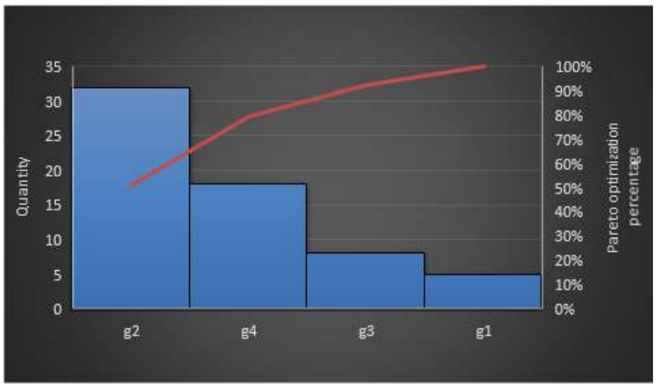

Fig. 3. The "Pareto optimal" distribution of elements of the dataset *g* in the network infrastructure

In all the above examples, the network infrastructure was Pareto optimized without taking into account the stability functions (only technical optimization).

Taking into account various stability functions, it is determined that the Pareto optimal network infrastructure can go into a state of instability.

Instability of the network infrastructure can be both hardware and software failures as a result of a random event.

Also, instability can be caused as a result of a successful attack on the network infrastructure, due to the presence of devices and software that did not pass the $g_i^{(2)}$ $criteria$ test.

Example 1

- stability test function $Q_1 = \frac{dx_i}{dt} = 1 + t$ −x;
- a detailed solution to the equation is given in [16];
- the solution is stable over the entire interval $t \longrightarrow \infty$

The random function $Q_1 = \frac{dx_i}{dt} = 1 + t$ −x is taken as a function of stability.

The stability function $Q_1 = \frac{dx_i}{dt} = 1 + t$ −x presented in Figure 4.

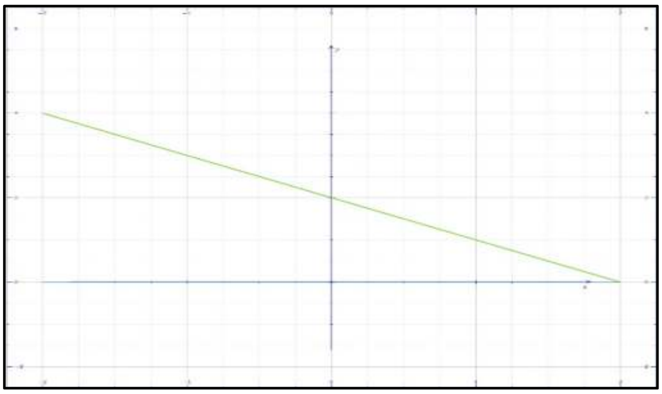

Fig. 4. The stability function $Q_1 = \frac{dx_i}{dt} = 1 + t$ −x graph

Result:

Any non-negative "Pareto optimal" solution for a given multicriteria dataset will be optimal.

Example 2

- stability test function $Q_2 = \frac{dx_i}{dt} = 1 - x^2(t)$
- a detailed solution to the equation is given in [16]

The solution $\varphi_i(t)$ is as follows.

- $\varphi_i(t) = 1$ network infrastructure is stable
- $\varphi_i(t) = -1$ network infrastructure becomes unstable

The stability function $Q_2 = \frac{dx_i}{dt} = 1 - x^2(t)$ graph presented in Figure 5.

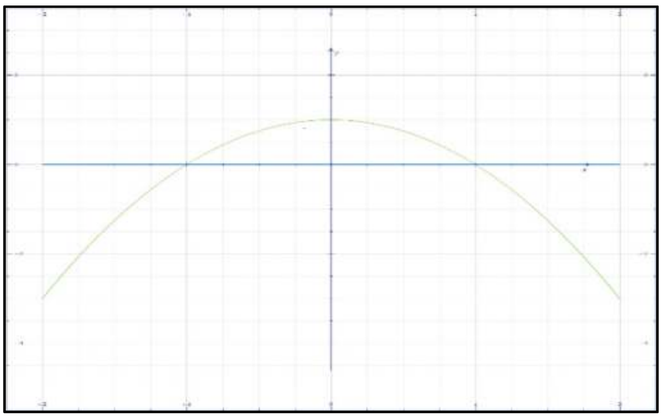

Fig. 5 The stability function $Q_2 = \frac{dx_i}{dt} = 1 - x^2(t)$ graph

Result:

The above calculations for the dataset *R,D,G* at the physical layer are described as follows.

Failure of an element from the data set *R ,D, G* ($\varphi_i(t) = -1$) leads to unstable operation of the network infrastructure (and the greater the level of influence of this element, the more unstable the network infrastructure becomes), although according to the graphs (Fig.1, 2, 3) the multicriteria change is Pareto optimal".

The solution $\varphi_i(t) = 1$ is a special case of the solution for the function Q given in Example 1.

VI. Conclusions

The article discusses the conditions for the stability of the network infrastructure in multicriteria optimization. Datasets R, D, G, divide each segment of the multicriteria optimization for subsets.The introduced concept of the "external stability function" makes it possible to assess the possibility of optimizing a specific dataset of stratified multicriteria datasets. The issue of tools for assessing and calculating multicriteria optimization is considered separately. A decision-making algorithm developed to optimize the multicriteria stratified network infrastructure. It is determined that an important condition for the stability of the network infrastructure in multicriteria stratification is the definition of the function (functions) of stability both for the local dataset and for the entire set of network infrastructure components.

The presented calculations prove that within the framework of multicriteria optimization of the network infrastructure, it is necessary to take into account the external function of stability, since not in all cases the "Pareto optimal" solution increases the stability of the system. The developed model makes it possible to assess the stability of the network infrastructure in terms of its multicriteria optimization.

A practical test of the model carried designing a network infrastructure for a heterogeneous network at the network core layer.

References

[1] Cisco network design. SEDP- Network Foundation Design [Online].Available:https://www.cisco.com/c/dam/en/us/td/docs/solutions/Enterprise/Small_Enterprise_Design_Profile/chap2sba.pdf

[2] W. Odom, *CCNA 200-301, Volume 1 Official Cert. Guide.* CCIE® NO 1624 EMERITUS. Library of Congress Control Number: 2019908180. Cisco Press. 2020

[3] W. Wilson, N. Rivera. *Cisco Networks. Engineers` Handbook of Routing, Switching, and Security with IOS, NX-OS, and ASA.* Second Edition, Apress®, 2021.

[4] V. D. Noghin. *Restriction of the Pareto set: an axiomatic approach*. M. FIZMATLIT, 2016.

[5] V. D. Noghin, What is the relative importance of criteria and how to use it in MCDM. - In "Multiple Criteria Decision Making in the New Millenium*", Proceedings of the XV International Conference on MCDM* (ed. by M. Kasalan, S. Zionts) in Ankara, Turkey (July, 2000), Springer, pp. 18-26, 2001.

[6] V. M. Bezruk, *Pareto-optimal solutions for multicriterial random signal recognition problems*, Radioelectronics. Informatics. Management, no.1.Neuroinformatics and Intelligent systems, 2005.

[7] V. D. Noghin., Multicriteria Choise Based on Fuzzy Information. *Scinentific and Techical Information Processing,* vol. 47, no. 5, pp. 32-37, 2020.

[8] F. Shamom, Z. Aziz, J. Liu, A. Martey. *Troubleshooting IP Routing Protocols* (CCIE® Professional Development). Cisco Press, 2002.

[9] *Essential Tools for OSI Internet*. RFC 1574. February 1994.

[10] *Host Extensions for IP Multicasting*. RFC 1112. August 1989.

[11] *Information technology-Open Systems Interconnection-Basic reference Model: The Basic Model. International Standart ISO/IEC-7498-1.* Second edition. Corrected and reprinted 1996-06-15.

[12] I. Kotenko, I. Saenko, D. Kotenko, *Methods and tools for attack modeling in large computer networks: state of the problem*. Artisle in SPIRAS proceedings. Issue 3(22). pp. 24-30, 2012.

[13] T. L. Saaty, *Multicriteria decision making. The analytic hierarchy process.* – Pittsburgh: RWS Publications, 1990.

[14] B. J. Hunt, M. M. Wiecek., C. S. Hughes, *Relative importance of criteria in multiobjective programming: A cone-based approach* // European J. of Operational Research, 2010.

[15] A. M. Lyapunov, "*The general problem of the stability of motion. // State publishing house of technical and theoretical literature"*. Moscow. Leningrad. 1950.

[16] Mathematical help Planet. [Online].Available:http://mathhelpplanet.com/static.php?p=ustoichivost-lyapunovu.

[17] V. D. Noghin, *Lectures on the theory stability of movement*. St.Petersburg. Publishing and Printing Association institutions of higher education. 2020.

[18] A. P. Ignatiev, Z. V. Smirnova. *Lyapunov stability: basic concepts and definitions.* Nauchnye vedomosti 15 (70), "Use of Pareto models in the formation of innovative activities of a telecommunications company." pp. 3-9, 2009.

[19] A. V. Lotov, V. A. Bushenkov, G. K. Kamenev, *Interactive decision maps, Approximation band visualization of Pareto frontier*. Boston, Kluver, 2004.

[20] B. Wolfson. O. Vlasov. *Flexible project and product management*. Peter publishing house. 2017.

[21] Olly IT. Information Technology. Official website :[Online].Available: https://www.olly.ru/blog/setevaya-infrastruktura/

[22] D. Teare, B. Vachon, R. Graziani. *Implementing Cisco IP Routing (Route). Foundation Learning Guide*. Cisco.press. 2015.

[23] *Guide for creation, selection, and registration of an Autonomous System (AS).* RFC 1930. March 1996.

[24] E. Tanenbaum. *Computer networks.* Series "Classics of computer sciense". Peter 2003.

[25] S.Harris, F.Maymi. *CISSP All-in-One Exam Guide.* 8th Edition. Publisher, McGraw-Hill, 2018.

[26] Departament of defence standart. *Defense Standard Department of Defense trusted computer system evaluation criteria «Orange book»*, December 26, 1985.

[27] M. Orlov. *Data Analysis and Intellectual Systems.An Algorithm for Multicriteria stratification*. Business Informatics no.4(30), pp.7-8, 2014.

[28] Olifer V.G. Olifer N.A. *Computer networks. Principles, technologies, protocols. 4th edition.* St. Petersburg 2010.

[29] IBM corporated official website software section: [Online].Available:https://www.ibm.com/analytics/spss-statistics-software

[30] R. William *Cisco OSPF Command and Configuration Handbook* (CCIE Professional Development) Cisco press. 2002.

[31] S. Halabi, D. McPherson. *Internet Routing Architectures,* Second Edition. Cisco press. 2000.

[32] *IP Security (IPsec) and Internet Key Exchange (IKE) Document Roadmap.* RFC 607. February 2011.