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Closed and mixed-type queuing models for structural control of complex technical systems

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***Abstract*—The presented paper discusses the problem of structural control for a complex technical system and its mathematical interpretation. The close and mixed exponential queuing model for the system dependability and performability analysis is constructed in the form of finite and infinite system of ordinary linear differential equations. In steady state, it is reduced to the system of linear algebraic equations.**

***Keywords*—Markov process, queuing model, reliability, maintenance renewal, replacement**

I. Introduction

The research topic belongs to the Mathematical Theory of Reliability (MTR), which applies the methods and models of Queuing Theory (QT) while investigating dependability and performability analysis of complex systems. We consider a multi-unit redundant system with repairable units, such as info-communication, computer and transportation networks, power and defense systems, etc. The problem is possible to formulate in terms of the practical problems of Queuing Theory, but we prefer to formulate it in terms of practical problems, for the solution of which, the necessity of our research has arisen. The research topic is significance within the network maintenance problem [1-6].

Info communication networks are an important attribute of modernity, so the problem of structural control of a complex technical system is very relevant, namely diagnostics, redundancy control and maintenance. Carrying out structural control means ensuring the maintenance of the existing structure of a controlled object - its elements, including connections and their modes of operation. Otherwise it is a compensation for structural disturbances (change of the system structure as a result of the failure of its elements). Disturbances can be of different types, we discuss structural disturbances, the compensation of which requires the organization of structural control [1-3].

We should mention here the high importance of open queuing models for dependability and performability analysis of modern territorially distributed networks. The fact is that for a long period of time in MTR and QT there was generally accepted the idea, that in problems of reliability and maintenance of redundant complex systems only finite-source (closed) queuing models were applicable.

This idea, however, is valid for classical machine maintenance (repairman, machine interference) problem, but for modern network maintenance problem open queuing models or mixed-type models are mainly applied. This is convincingly verified by experts from Georgian Technical University in their publications for last years [6-8], [12-16].

II. Problem Statement

In the classical theory of reliability, the repairment problem is formulated as follows: Suppose we are given $\boldsymbol{m}$ identical, stochastically independent units and their identical $\boldsymbol{n}$ redundant units $(m > n)$. Assume that each is by some distribution of lifetime. Also, suppose our repair channel can repair $\boldsymbol{k}$ $(k < n)$ element. If the repair channel is busy, each new failed unit will move to the queue and wait for the repair channel to be released.

The failed main unit must be replaced by operative redundant one. Thus, if at failure moment there is a free redundant unit in the system, its replacement operation will happen. The failed units, both main and redundant ones, must be repaired. The repaired unit is supposed to be identical to the new one.

We assume that the repair times are also independent, identically distributed random variables with some repair time distribution. The diagram in Fig.1 will be helpful in explaining various models of repairman problems.

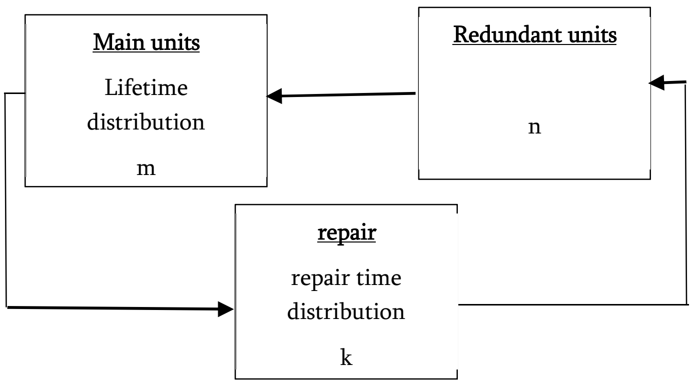

Fig. 1. Diagram illustrating repairman (maintenance) problems. From Richard E. Barlow, Frank Proschan, Mathematical Theory of Reliability. SIAM, 1996

In this case, we construct the mathematical model. Let the failure intensity of one main element be α, while the failure intensity of one Redundant element is β. Repair time is an exponentially distributed random variable with parameter λ.

Let $P_i(t)$ denote the probability that the number of failed elements is i in the system at time t.

Proceeding in the usual way, we can set up the basic difference equations, which relate the probability of being in a certain state at time t +Δt to the probabilities of being in various states at time t. From these difference equations we obtain the finite systems of the ordinary linear differential equations.

We have finite system of ordinary differential equations.

$$\frac{dP_0(t)}{dt} = -m\alpha P_0(t) - n\beta P_0(t) + \lambda P_1(t) \quad (1)$$

$$\frac{dP_1(t)}{dt} = -m\alpha P_1(t) - (n-1)\beta P_1(t) + (m\alpha + n\beta)P_0(t) + 2\lambda P_2(t) \quad (2)$$

$$\frac{dP_i(t)}{dt} = (-m\alpha - (n-i)\beta - i\lambda)\, P_i(t) + (m\alpha + (n-i+1)\beta)P_{i-1}(t) + (i+1)\lambda P_{i+1}(t) \qquad 0 < i \le k \quad (3)$$

$$\frac{dP_i(t)}{dt} = (-\mathrm{m}\alpha - (\mathrm{n}-\mathrm{i})\beta - k\lambda)P_i(t) + (m\alpha + (\mathrm{n}-\mathrm{i}+1)\beta)P_{i-1}(t) + k\lambda P_{n+1}(t), \qquad k < i < n \quad (4)$$

$$\frac{dP_n(t)}{dt} = (-m\alpha - k\lambda)P_n(t) + (m\alpha + \beta)P_{n-1}(t) + k\lambda P_{n+1}(t) \quad (5)$$

$$\frac{dP_i(t)}{dt} = (-(\mathrm{m}-\mathrm{i})\alpha - k\lambda)P_i(t) + (\mathrm{m}-\mathrm{i})\,\alpha P_{i-1}(t) + k\lambda P_{i+1}(t), \qquad n < i < n+m \quad (6)$$

$$\frac{dP_{n+m}(t)}{dt} = -k\lambda P_{n+m}(t) + \alpha P_{n+m-1}(t) \quad (7)$$

Consider a queuing system operates in a stationary state, denote $\mathrm{Pi} = \lim_{t\to\infty} \mathrm{Pi}(t)$ letting t =→ ∞ in (1) -(7), we get a system of linear algebraic equations together with the normalizing condition:

$$(m\alpha + n\beta)P_0 = \lambda P_1 \quad (1a)$$

$$(m\alpha + (n-1)\beta)P_1 = (m\alpha + n\beta)P_0 + 2\lambda P_2 \quad (2a)$$

$$(m\alpha + (n-i)\beta + i\lambda)\, P_i = (m\alpha + (n-i+1)\beta)P_{i-1} + (i+1)\lambda P_{i+1} \qquad 0 < i \le k \quad (3a)$$

$$(\mathrm{m}\alpha + (\mathrm{n}-\mathrm{i})\beta + k\lambda)P_i(t) = (m\alpha + (\mathrm{n}-\mathrm{i}+1)\beta)P_{i-1}(t) + k\lambda P_{n+1}(t), \qquad k < i < n \quad (4a)$$

$$(m\alpha + k\lambda)P_n(t) = (m\alpha + \beta)P_{n-1}(t) + k\lambda P_{n+1}(t) \quad (5a)$$

$$\big((\mathrm{m}-\mathrm{i})\alpha + k\lambda\big)P_i(t) = (\mathrm{m}-\mathrm{i})\,\alpha P_{i-1}(t) + k\lambda P_{i+1}(t), \qquad n < i < n+m \quad (6a)$$

$$k\lambda P_{n+m} = \alpha P_{n+m-1} \quad (7a)$$

$$\sum_{i=0}^{n+m} P_i = 1$$

III. Mathematical Model for m=∞

The classical mathematical theory of reliability does not allow to solve the problems of dependability and performance analysis of large scale territorially distributed networks. For these purposes, further extending, development and deepening of classical repairman (maintenance) models are necessary. We give an explanation of a specific example: the number of Radio Base Station (RBS) in modern mobile communication networks may be hundreds, thousands and more. That means that in mathematical models we can consider the set of RBS as an infinite source of failures. Due to the same factor, we can consider the total failure rate to be constant. Consequently, we will have a Poisson stream of requests to maintenance facilities. As it is known, this is very important for the construction of suitable mathematical models and also for their investigation. Figure 2 shows interpretation of the repairman problem considering this factor.

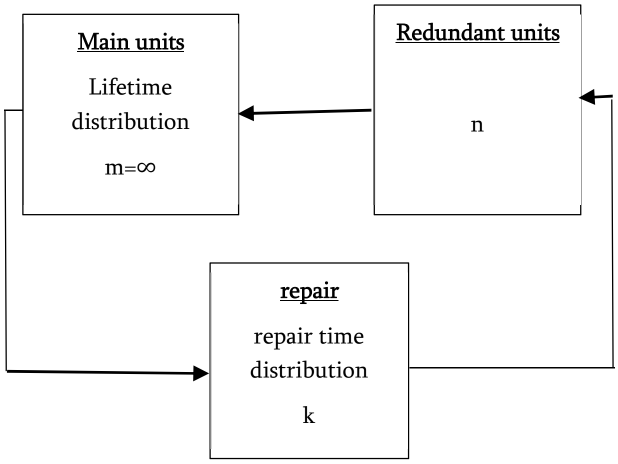

Fig. 2 Diagram illustrating the non-classical repairman problem

In this section, we consider the case, where ***m*** is a large number (in practice it might be tens, hundreds, thousands and more), and we will suppose that we have an infinite source of requests and will get an open queuing system.

We construct the mathematical model for the case where m = ∞ , n is arbitrary. The total failure intensity of main elements is α, while the failure intensity of one Redundant element is β. Repair time is an exponentially distributed random variable with parameter λ.

We obtain the infinite systems of the ordinary linear differential equations.

$$\frac{dP_0(t)}{dt} = (-\alpha - n\beta)P_0(t) + \lambda P_1(t) \quad (8)$$

$$\frac{dP_i(t)}{dt} = (-\alpha - (n-i)\beta - i\lambda)\, P_i(t) + (\alpha + (n-i+1)\beta)P_{i-1}(t) + (i+1)\lambda P_{i+1}(t), \qquad 0 < i \le k \quad (9)$$

$$\frac{dP_i(t)}{dt} = (-\alpha - (\mathrm{n}-\mathrm{i})\beta - k\lambda)P_i(t) + (\alpha + (\mathrm{n}-\mathrm{i}+1)\beta)P_{i-1}(t) + k\lambda P_{n+1}(t), \qquad k < i \le n \quad (10)$$

$$\frac{dP_i(t)}{dt} = (-\alpha - k\lambda)P_i(t) + \alpha P_{i-1}(t) + k\lambda P_{i+1}(t), \qquad i > n \quad (11)$$

Denote $\mathrm{Pi} = \lim_{t\to\infty} \mathrm{Pi}(t)$ Letting t =→ ∞ in (1) - (4), we obtain an infinite system of linear algebraic equations with respect to P_{i}.

$$(\alpha + n\beta)P_0 = \lambda P_1 \quad (8a)$$

$(\alpha + (n - i)\beta + i\lambda)\, P_i = (\alpha + (n - i + 1)\beta)P_{i-1} + (i + 1)\lambda P_{i+1} \qquad 0 < i \le k \qquad (9a)$

$(\alpha + (\mathrm{n} - \mathrm{i})\beta + k\lambda)P_i = (\alpha + (\mathrm{n} - \mathrm{i} + 1)\beta)P_{i-1} + k\lambda P_{i+1} \qquad k < i \le n \qquad (10a)$

$(\alpha + k\lambda)P_i = \alpha P_{\mathrm{i}-1} + k\lambda P_{n+1} \qquad i > n \qquad (11a)$

$\sum_{i=0}^{\infty} P_i = 1$

We get an infinite system of linear algebraic equations. The existence of a solution to this system of equations, the uniqueness, and the task of constructing an approximate solution require special consideration, and in this article we will not touch on this case, we will refer only to the relevant literature [9]-[11].

IV. Infinite Number of Service Channels

Organizations around the world are looking for new approaches to maintain or/and increase their competitiveness. Maintenance outsourcing as one of the methods to minimize operating cost is sometimes an alternative [17]. Consider the case where the number of service channels is infinite –by using outsourcing (the source is also infinite), any application immediately starts the service, i.e. there are no queues. The intensity of the failure is α, the intensity of the repair is μ. The system of differential equations will have the form:

$$\frac{dP_0(t)}{dt} = -\alpha P_0(t) + \mu\, P_1(t) \qquad (12)$$

$$\frac{dP_1(t)}{dt} = -(\alpha + \mu)P_1(t) + \alpha\, P_0(t) + \mu\, P_2(t) \qquad (13)$$

$$\frac{dP_i(t)}{dt} = -(\alpha + i\mu)\, P_i(t) + \alpha\, P_{i-1}(t) + (i + 1)\, \mu\, P_{i+1}(t) \qquad i > 1 \qquad (14)$$

letting t =→ ∞ in (12) -(14), we get an infinite system of linear algebraic equations:

$$\alpha P_0 = \mu P_1 \qquad (12a)$$

$$\alpha P_1 = 2\mu P_2 \qquad (13a)$$

$$\alpha P_i = \mathrm{i}\mu P_{\mathrm{i}+1} \qquad i > 1 \qquad (14a)$$

From where we get

$$P_1 = \rho P_0 \qquad (12b)$$

$$P_2 = \frac{1}{2}\rho^2\, P_0 \qquad (13b)$$

$$P_i = \frac{1}{\mathrm{i}\,!}\rho^i\, P_0\ ; \qquad \rho = \frac{\alpha}{\mu} \qquad (14b)$$

V. Conclusion

It is known that the instantaneous production of an object and the quality of the output products mainly depend on the quality of its coordinate and parametric control. But the integrated solution of the facility is determined by its reliability and consequently by the level of its structural control organization. It can be said that the degree of coordinate and parametric control is revealed instantly (high accuracy, agility, etc.), And the quality of structural control - over a relatively long interval of operation. In other words, the structural control of complex systems is a content-intensive strategic problem, and its solution requires the development of appropriate, specific, methods and tools. The implementation of the principles of quality assurance and reliability of complex technical facilities allows the effectiveness of technical systems to be assessed by analytical methods during both design and operation.

We discussed the close and mixed exponential queuing model for the system where incoming stream is the Poisson flow. Under such conditions, the system is described as "birth-death" Markov process. We constructed in the form of finite and infinite system of differential equations.

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