The Systems with Reconfigurable Structure Based on Multi-Functional Elements

Sergo Tsiramua
University of Georgia
Tbilisi, Georgia
e-mail: s.tsiramua@ug.edu.ge

Hamlet Meladze
Muskhelishvili Institute of Computational
Mathematics of Georgian Technical University
Tbilisi, Georgia
e-mail: h_meladze@hotmail.com

Tinatin Davitashvili
Ivane Javakhishvili Tbilisi State University
Tbilisi, Georgia
e-mail: tinatin.davitashvili@tsu.ge

Irakli Basheleishvili
Akaki Tsereteli State University
Kutaisi, Georgia
e-mail: basheleishvili.irakli@gmail.com

***Abstract*—Multi-functional elements are a special class of elements, the reliability model od which differs from the classical two-pole "on-off" model. A multi-function element can have partly false states in addition to non-false and false states. The multi-functionality of the elements leads to the formation of flexible, adaptable systems with reconfigurable structure, in which, in case of partial failure of the element, it is possible to continue the successful functioning of the system by redistributing the functions between the elements. In this paper, the properties of multi-functional elements and systems, assembled on their basis, reliability models and issues of optimal reconfiguration are discussed.**

***Keywords*— Multi-functional elements, reconfigurable systems, flexibility, maneuverability, reliability.**

I. Introduction

The multi-functionality of elements leads to the formation of flexible, structurally re-configurable, adaptable systems, in which, in case of partial failure of the element, it is possible to continue the successful functioning of the system by redistributing functions between the elements and exchanging them. Such properties can be observed in completely different types of systems, for example, technical, energy, automation, production, organizational, human-machine and other types of systems. In case of technical systems, it is possible to distinguish robotic systems equipped with multi-functional robots, computer clusters, multi-processor computers, multi-core processors, cloud server systems, etc. [1], [2]. In organizational, industrial or man-machine systems the complex production brigades, dispatch services, transport crews, project groups, sports teams, etc., can be composed of multi-functional specialists [3], [4].

Let us consider the properties of multi-functional elements and systems constructed on their basis, reliability models and issues of their optimal reconfiguration.

II. Multi-functional Elements

A multi-functional element (MFE) is an element with functional redundancy, which has the ability to perform at any time moment t any one function f from the set of system's functional resources (functional capabilities) $F_a = \{f_e | e \in [1,k]\}, k > 1$ [3].

Only one state out of the number of all possible failures (2^k) regarding the separate functions of the MFE corresponds to the complete failure of the element, when it loses the ability to perform all functions. The rest of the states $(2^k - 1)$, when the element can perform at least one function, correspond to the element's performance states. However, in case of partial failure of the element in relation to the function assigned to it, when it still remains in a working condition, it may not be enough for the system to continue successful functioning if the lost function is not compensated by another element.

Depending on the functional capacity, the MFE can be dual-functional for a given system $(k = 2)$, tri-functional $(k = 3)$ and so on, k-functional $(k > 1)$. When $k = 1$, we are dealing with a classical type single-function element, which in a given system can perform only one function, assigned to it, out of system's functions set F, and it can be in an operating or failure mode.

Based on this the number of functions m imposed on the system A from the set F, MFE can be functionally complete $(k = m)$ or incomplete $(k < m)$ for the given system.

If MFE a has an ability to perform any function $F_a = F$ from the set of functions $F = \{f_j | j \in [1,m]\}$, assigned to the system A, we call such an element a functionally complete element.

If the MFE a can perform some part of the functions $F_a \subset F$ from the functions assigned to the system A, we call such MFE a functionally incomplete element with respect to the given system.

From the point of view of failure, it is possible to have a complete failure of the MFE, when the element loses the ability to perform the function assigned to it and at the same time loses all the functional resources that it has in relation to the given system.

On the other hand, in the process of functioning, the partial failure of the element is possible, when the element loses the ability to perform the function assigned to it, but retains the ability to perform other functions assigned to the system.

In the first case, i.e., in the case of complete failure, the element completely fails and in order to continue the successful operation of the system, it is necessary to restore the element or replace it with a backup element in case of non-recoverable failure.

In the second case, when we have a partial failure of multi-functional element and it loses the ability to perform the assigned function, it can switch to another function, and the performance of its lost function can be continued by another multi-functional element of the system. In such a case, the successful functioning of the system is ensured by the inter-changebility of elements. Thus, in the case of partial failure of MFE, when the failure occurs only with respect to the function $f_i \in F_a$ which is in the process of execution, the MFE is switched in the supposed time interval to the other function $f_j \in F_a$ from the set of the functions $F_a = \{f_e | \, e \in [1, k]\}$ and the missing function f_i is being performed by another MFE of the given system, which previously performed the function f_j (the mutual replacement of elements occurs).

The goal of our research is the second case, since the reliability of systems based on multi-functional elements considering the partial failure of the elements, is less studied and investigated. Accordingly, the issues of modeling, synthesis, reliability and design of such systems are less covered in scientific publications. Without modeling and quantitative assessment of reliability, it is impossible to design systems with high reliability based on multi-functional elements.

III. Logical Models of Systems with Multi-functional Elements

The structure of the system created on the basis of MFE is determined by the composition of the elements, their inter-relationships, the functional resources (functional capabilities) of the elements and the scheme of distribution of functions between the elements. Thus, the main parameters of the structure of the systems composed on the basis of MFE are:

n – number of elements in a system $A = \{a_1, a_2, ..., a_n\}$;

m – number of functions $F = \{f_1, f_2, ..., f_m\}$ imposed on the system A;

k_i – the number of functional resources of i-th MFE;

k_Σ – the sum of the functional resources of all MFEs of the system (the number of functional resources of the system);

δ_S - scheme of distribution of functions between elements.

The characteristics of assessment criteria of system reliability - the probability of reliable operation, flexibility, viability, resistance to failure - uniquely depend on the structural parameters and their interrelationship.

When modeling the reliability of systems designed on the basis of MFEs, it is important to consider the operation mode of the system.

Consider such a mode of operation of the system when: 1) In the system, simultaneously, in parallel mode, all functions assigned to the system, which are distributed among multi-functional elements are performed; 2) Each MFE can perform only one function from its set of functional resources at each moment of time.

When the system consists of MFEs, the system working in such a parallel mode has a flexible structure not based on the reserve elements, but based on a functional redundancy of MFEs. If a partial failure of any MFE occurs in relation to the assigned function, such a replacement of the elements will be possible (rearrange the system structure) when the condition of simultaneous performance of all the functions assigned to the system is restored. It should also be noted here that the effective functioning of such systems depends not only on n, m, k_Σ and on k parameters, but also on the δ_S scheme of distribution of functions between MFEs.

Thus, it is assumed that by the system $A = \{a_i | \, i \in [1, n]\}$ the function $F = \{f_j | \, j \in [1, m]\}$ succeeds, if in a given time interval T all functions f_j, $j \in [1, m]$ from the set F of assigned to the system functions are performed, provided that any MFE a_i of system A at any time moment $t_\tau \in T$ performs only one function from the set of its functional resources $F_a = \{f_e | \, e \in [1, k]\}$, $k > 1 \; (F_a \subseteq F)$.

In the reliability model of systems with a reconfigurable structure, all the ways of successful operation of the system should be considered [5]. When system A consists of MFEs, then the function F can be performed by different distribution of functions between MFEs. In order to build a reliability model, it is advisable to describe using logical functions the ways of successful functioning of the system and the condition of the its operability.

Let us build the logical matrix $B(m \times n) = [a_i(f_j)]$ of functional resources of the system, in which

$$a_i(f_j) = \begin{cases} 1, \text{if MFE } a_i \text{ can perform a function } f_j, \\ 0, \text{if MFE } a_i \text{ can't perform a function } f_j. \end{cases}$$

In the general case, when $n \geq m = k_i > 1$, $i \in [1, m]$, that is the system is composed of functionally complete MFEs, the shortest ways of successful functioning of the system can be written using the following conjunctions:

$$S_q = a_{i1}(f_{j1}) \wedge a_{i2}(f_{j2}) \wedge \cdots \wedge a_{im}(f_{jm}), \qquad (1)$$

where

$i_1 = 1, 2, ..., n - m + 1;$

$i_2 = i_1 + 1, i_1 + 2, ..., n - m + 2;$

$\cdots\cdots\cdots\cdots$

$i_m = i_{m-1} + 1, i_{m-1} + 2, ..., n;$

$j_1, j_2, ..., j_m \in [1, m]$; $j_1 \neq j_2 \neq ... \neq j_m$; $q \in [1, N_S]$.

N_s is an indicator of system flexibility, which represents the number of options for the distribution of functions between MFEs, that is, it shows the number of the shortest ways of functioning of the system. The index of flexibility of the adjustable system N_s, $n > m = k_i > 1$, $i \in [1, m]$, which is described by $B(m \times n)$ rectangular matrix, is calculated by the formula:

$$N_s = per\{B(m \times n)\} = n!/(n-m)!, \quad (2)$$

where per is a permanent of the matrix $B(m \times n)$.

It follows from (1) that each shortest way of functioning of the system represents the conjunction of such elements of matrix $B(m \times n)$ that are located in different rows and columns of the matrix. The system operability condition is described by the disjunction of the shortest paths of operation:

$$F_A[a_i(f_j)] = \bigcup_{q=1}^{N_S} S_q. \quad (3)$$

In the special case when $n = m = k_i > 1$, $i \in [1, m]$ and the matrix of functional resources of the system is a square unit matrix, the shortest paths for the successful functioning of the system can be written by the following logical function:

$$S_q = a_1(f_{j1}) \wedge a_2(f_{j2}) \wedge ... \wedge a_n(f_{jm}), \quad (4)$$

where $j_1, j_2, ..., j_m \in [1, m]$; $j1 \neq j2 \neq ... \neq jm$; $q \in [1, N_s]$.

$$N_s = per\{B(m \times n)\} = n!. \quad (5)$$

The condition of system operability in this case is also described by Formula (3).

In both considered cases, when the system is composed of functionally complete elements ($m = k_i > 1$, $i \in [1, m]$), the index of flexibility of the system (the number of shortest paths of successful functioning) increases as factorial with the increase of N_s, m and n ($N_s = n!/(n-m)!$ or $n!$). When the system is composed of a functionally incomplete MFE ($m > k_i > 1$, $i \in [1, m]$), obviously the number of ways of functioning is less than $n!/(n-m)!$ or $n!$ and depends on the scheme of distribution of functions between the MFEs δ_S. Although a manual modeling of the ways of functioning of the system is also difficult in this case. In both cases, it is advisable and even necessary to model the ways of functioning using a logic model (1) on a computer.

The mode of operation discussed above characterizes multi-core processors operating in parallel computing mode and multi-processor computers, robotic systems and "Human-Machine" crews of transport systems, sports teams, etc.

Now let's consider such a case of system performance condition when: 1) All functions assigned to the system, which are distributed among multi-functional elements, are performed in sequential or mixed (parallel-sequential) mode; 2) Each MFE can perform only one function from its set of functional resources at each moment of time.

In such a case, when the system is constructed with functionally complete MFEs $n \geq m = k_i > 1$, $i \in [1, m]$, the shortest ways of functioning are described by the following logical function:

$$S_q = a_{i1}(f_1) \wedge a_{i2}(f_2) \wedge ... \wedge a_{in}(f_m), \quad (6)$$

where $i_1, i_2, ..., i_n \in [1, n]$; $q \in [1, N_S]$,

$$N_S = n^m. \quad (7)$$

Obviously, when the system is composed of functionally incomplete elements, then in the matrix of functional resources of the system $B(m \times n)$ in addition to 1s, there are also 0s, and the number of ways of functioning is less than n^m.

As in the case of the parallel mode, the condition of operability of the systems operating in the sequential mode is also described by the disjunction of the shortest paths of operation. It should be noted that under the given conditions the system can function successfully even when n^m. E.g., such class of systems includes a project group that involves multi-functional specialists who, at a certain interval of time, perform sequentially the tasks provided by the project.

IV. Optimal Management of Systems Reconfiguration

As we can see, there is a redundancy of ways of successful operation in the reconfigurable systems constructed on the base of MFEs in both considered working (parallel and sequential) modes. Such systems have high indicators of flexibility, which indicate high potential opportunities for restructuring (reconfiguration of the structure). Accordingly, the maneuverability of the system is increased, which characterizes the process of redistribution of functions between elements when fixing partial failure of an element. In this case, a comparative analysis of system flexibility and maneuverability is appropriate.

Flexibility of the system structure is a number of the shortest ways of functioning of the system (or the number of variants for the distribution of functions between the elements of the system), which is calculated by formulas (2), (5) and (7) in the case of functionally complete MFEs for the considered modes of operation.

Maneuverability is the ability of a system to rearrange its structure to continue successful operation in response to total or partial failure of an element.

Maneuvering is a process of reconfiguration, when, in case of element failure, the system structure is rearranged by replacing or interchanging elements (reconfiguration).

The number of maneuvering cases N_τ when the system is operating in parallel mode is minimal when there is a sequential loss and depletion of functional resources by any i-element with minimal functional resources. In this case, the system's functional resource matrix will have the following state:

$$a_i(f_1) = a_i(f_2) = ... = a_i(f_{min(k)}) = 0, i = const. \quad (8)$$

The number of maneuvers N_τ is minimal even when all elements of the system consistently lose the ability to perform the same function. In this case, the system's functional resource matrix will have the following state:

$$a_1(f_j) = a_2(f_j) = ... = a_n(f_j) = 0, j = const. \quad (9)$$

The number of maneuvers N_τ is maximal when various functions are successively lost by the different elements, and the moment in matrix of functional resources occurs when

$$a_{i1}(f_{j1}) = a_{i2}(f_{j2}) = ... = a_{im}(f_{jm}) = 0, \quad (10)$$

where $i_1 \neq i_2 \neq ... \neq i_m$, $j_1 \neq j_2 \neq ... \neq j_m$. In such a case, the system has a maximum number of reconfiguration variants.

In the general case, when $n \geq m \geq k$, $\max(N_\tau) = k_\Sigma - 2n + 1$; in a private case, when $n > m = k$, $\max(N_\tau) = nm - 2n + 1$; In case when $n = m = k$, $\max(N_\tau) = (n-1)^2$. Thus, the index of maneuverability of the system, which is expressed in the number of performed maneuvers, depends on the sequence of loss of functional resources by the elements and varies in the range $[\min(k_i) - 1, k_\Sigma - 2n + 1]$.

The flexibility of the system structure can be determined in advance during the system design process depending on whether the system is assembled from functionally complete or incomplete MFEs. As for maneuvering, it is an fuzzy process that can take place in case of partial or complete failure of the element. Predicting this in advance is associated with various difficulties, and in some cases is even impossible.

At the stage of designing the system, the tasks of the initial optimal distribution of functions among the elements, and at the stage of operation, in the case of fixing the failure of the element, the tasks of optimal reconfiguration of the systems are arising. In order to implement the mentioned tasks, it is necessary to move from the logical description of the functional resources of the system to the probabilistic description of the functional capabilities. Logical $(0, 1)$ matrix of system functional resources $B(m \times n) = [a_i(f_j)]$ should be replaced with the probability matrix $P(m \times n) = [p_i(f_j)]$, where $p_i(f_j)$, $i \in [1, n]$, $j \in [1, m]$, is the probability of performance the j–function by i-element without failure. Since MFEs belong to the class of multi-pole elements, Fuzzy Logic methods can be used to obtain estimations for $p_i(f_j)$ [6]:

$$p_i(f_j) = \begin{cases} 0 < p_i(f_j) < 1, \text{when } a_i \text{ can perform a function } f_j, \\ 0, \text{when } a_i \text{ can't perform a function } f_j. \end{cases}$$

Accordingly, the shortest ways of functioning of the system can be described using the following probabilistic representation:

$$P_F(S_q) = p_{i1}(f_{j1}) \times p_{i2}(f_{j2}) \times ... \times p_{im}(f_{jm}), \qquad (11)$$

where $i_1 \neq i_2 \neq ... \neq i_m$, $j_1 \neq j_2 \neq ... \neq j_m$, $q \in [1, N_S]$.

Based on the fact that in most cases, the probabilities of operation of MFEs differ for individual functions, which means that it is important to which way the system starts functioning and which way it continues to function after reconfiguration. If we know the numerical values of $P_F(S_1), P_F(S_2), ..., P_F(S_{N_S})$, we will be able to make an optimal rearrangement of the structure, for which we should rank $P_F(S_q)$ in descending order. In the process of forming a reconfigurable system, it is advisable to redistribute functions between MFEs with $\max P_F(S_q)$.

If we consider that the number of ways of successful operation of the system increases as factorial, with respect to the growth of parameters n, m and k, the need for optimal control of reconfiguration becomes clear. In order to automate and optimally reconfigure the described process, it is possible to use a modernized version of the target task:

$$\begin{gathered} F_A[a_i(f_j)] = \prod_{i=1}^{n} \prod_{j=1}^{m} p_i(f_j)x \rightarrow \max, \\ \sum_{i=1}^{n} x_{ij} = 1, \; j \in [1, m], \; \sum_{j=1}^{m} x_{ij} = 1, \; i \in [1, n]. \end{gathered} \qquad (12)$$

In contrast to the classical model of optimal destination, in the proposed model, the double sum is replaced by a double product in order to obtain a probabilistic value of estimation.

At the initial stage, at the stage of assembling the reconfigurable system with MFEs, when $n > m = k_i$, $i \in [1, n]$, using the model (12) the optimal selection of m from n elements is performed and the optimal distribution of functions between the elements are carried out. When $n = m = k_i$, $i \in [1, n]$, the optimal distribution of functions between elements is carried out using the model (12). In case of partial failure of the MFE in the process of functioning of the system, by entering 0 in the appropriate place in the matrix of functional resources and using the model (12), redistribution of functions between elements (interchange of elements) and optimal reconfiguration is performed. It should be noted that similar processes take place in parallel computing systems as well [7].

V. Conclusion

In the general case, when $n \geq m \geq k_i$, $i \in [1, n]$, the probability of successful operation of the reconfigurable system depends on the scheme of distribution of functions f_j among the MFEs both in the process of system design and operation. Using model (12), based on the evaluations of $P_F(S_q)$, it is possible to optimally design the system and optimally manage the reconfiguration process.

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